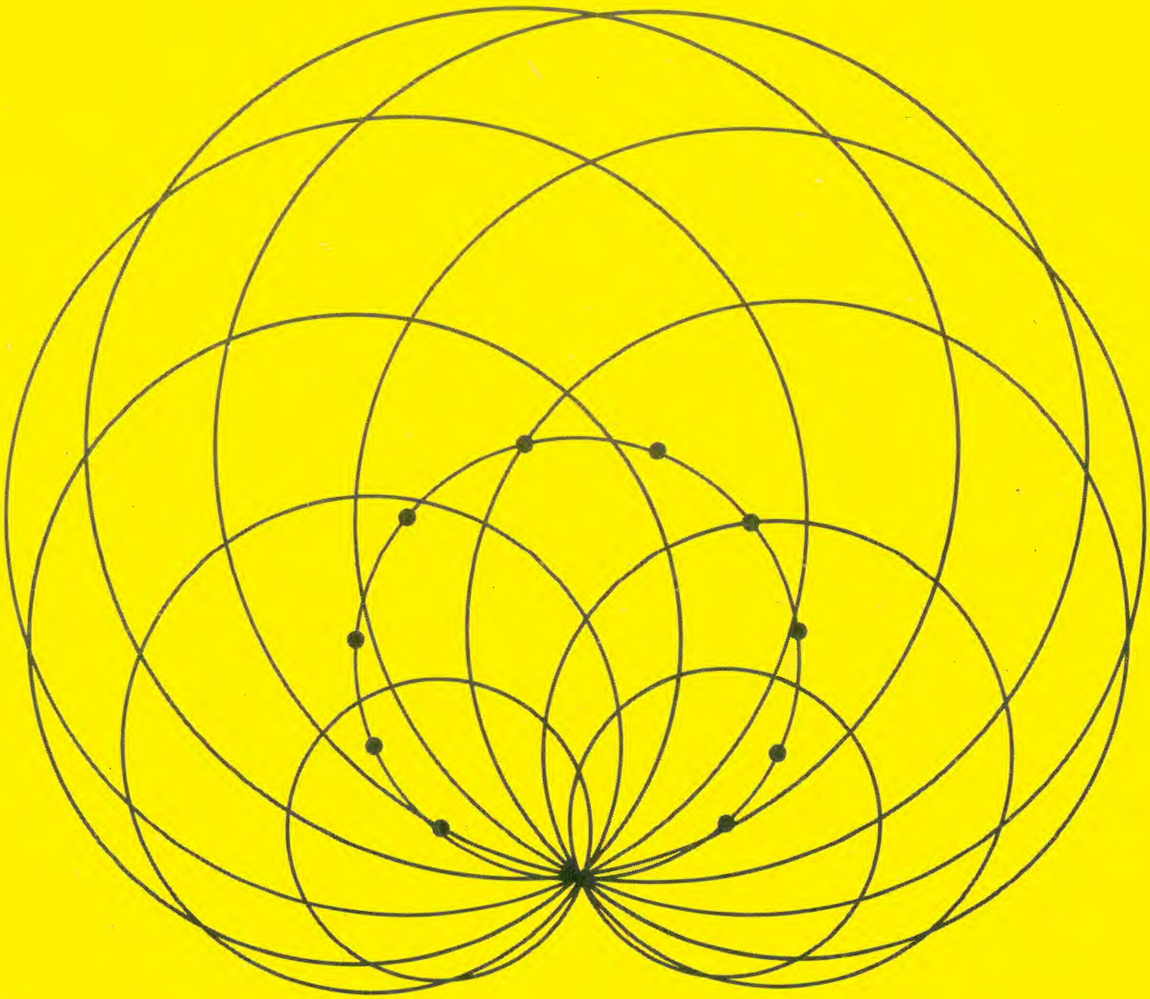
M·A·T·H·S INVESTIGATIONS

PUPILS' WORKSHOP



DAVID KIRKBY and PETER PATILLA

GRIDS

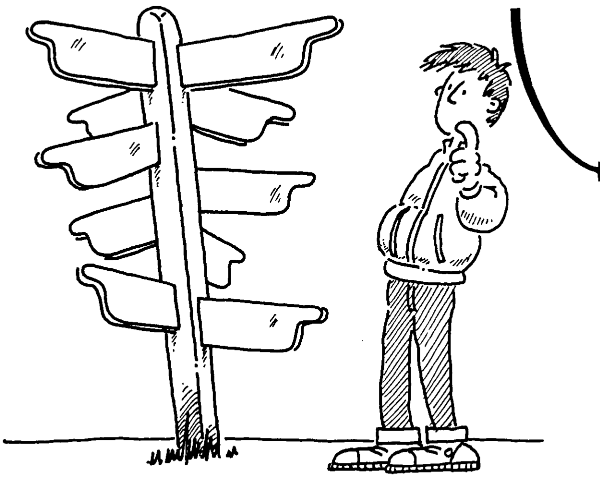
GRIDS

ROUTE 22

1 cm squared paper

Choose a target number – 22

Choose a starting position:



6	3	5	2	1
3	1	4	6	2
5	6	2	4	5
2	5	3	1	6
6	2	4	3	4

Find a route which totals 22.
Horizontal and vertical moves are allowed.

6	3	5	2	1
3	1	4	6	2
5	6	2	4	5
2	5	3	1	6
6	2	4	3	4

This is a five-stage route.
[5 + 6 + 1 + 4 + 6 = 22]

6	3	5	2	1
3	1	4	6	2
5	6	2	4	5
2	5	3	1	6
6	2	4	3	4

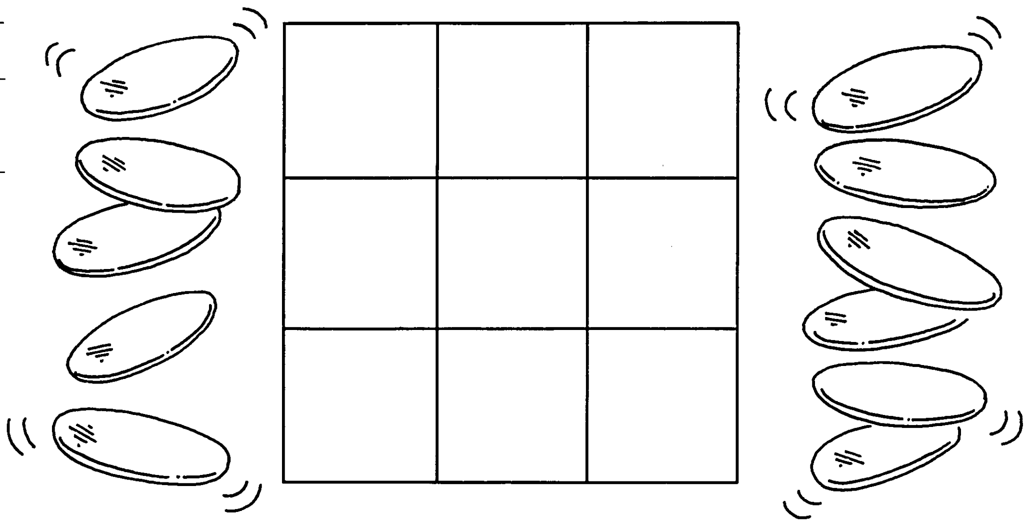
This is a six-stage route.
[5 + 3 + 6 + 3 + 1 + 4 = 22]

Investigate other routes for a target of 22.
Try different starting positions.
Try different targets.

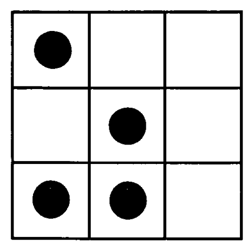
LINES OF THREE

Counters

Squared paper



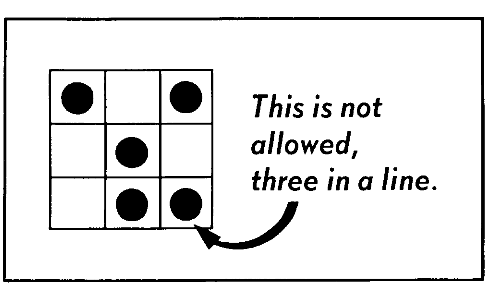
Here are four counters arranged on the grid:



Lines of three counters, horizontally, vertically or diagonally are not permitted.

Can you find other arrangements of four counters?

Try using five counters.
Investigate for different numbers of counters.

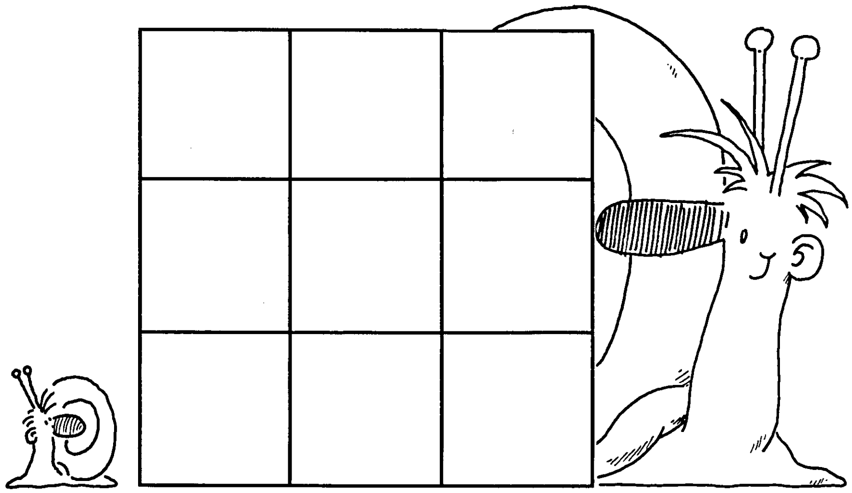


GRIDS

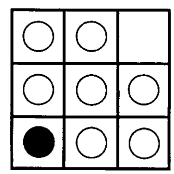
SNAILS

Counters

1 cm or 2 cm squared paper

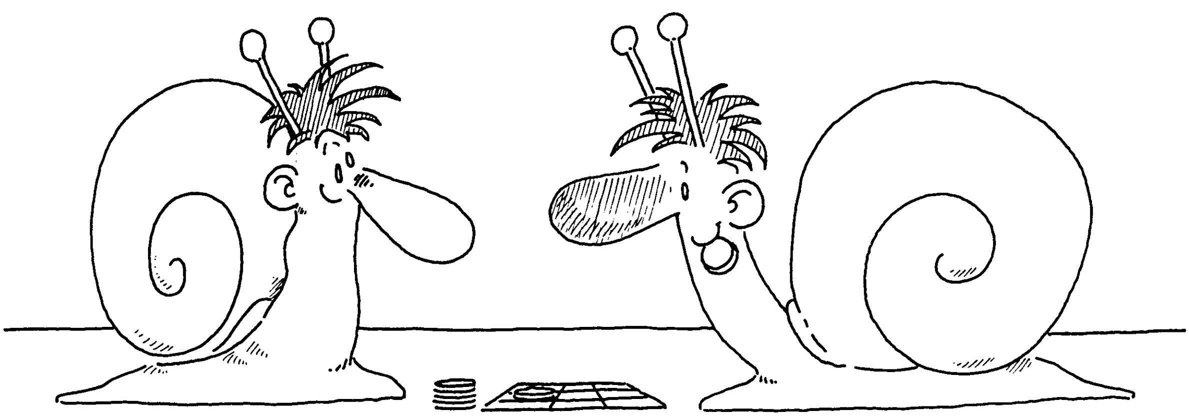


Place eight snails on the 3 × 3 grid.
The chief snail is a different colour and
is placed in the bottom left-hand corner.



Snails can only glide into an adjacent empty square.
They cannot glide diagonally.

Try to move the chief snail into the top right-hand corner.
Investigate the minimum number of moves required.



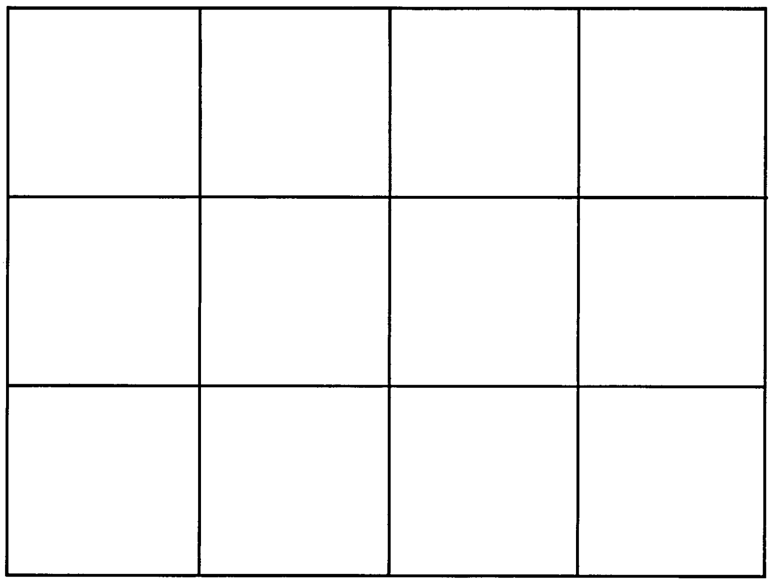
GRIDS

EVEN LINES

Counters

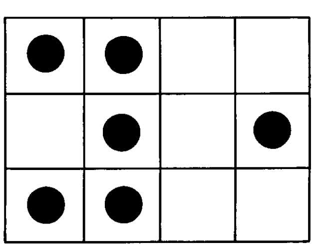
Squared paper

Can you place six counters on this 3 × 4 grid?
All rows and columns must contain an **even** number of counters.
No square may contain more than one counter.



Investigate different possible arrangements.
Can you place four counters, or five counters?

Try for other numbers of counters.

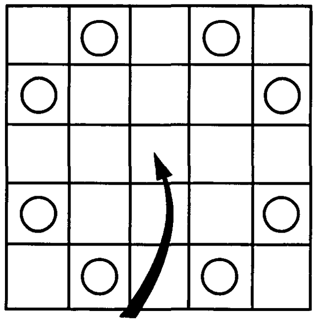


This is not allowed.

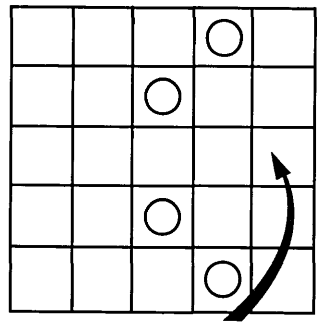
KNIGHT SHIFT

Squared paper

Counters



A knight placed here can jump to any of these eight squares.

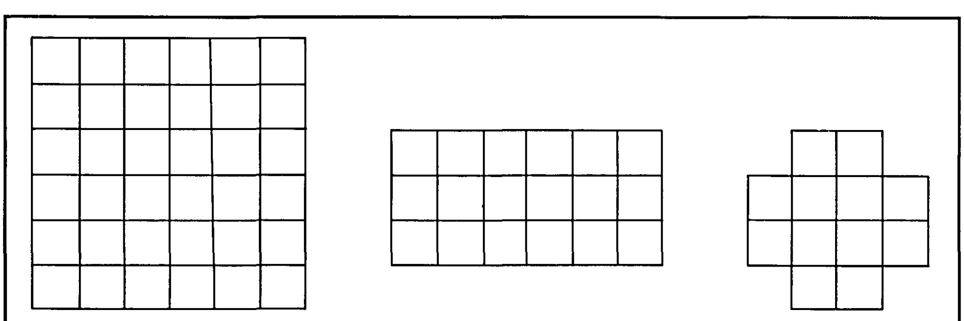


A knight placed here can jump to any of four squares.

Record the results like this:

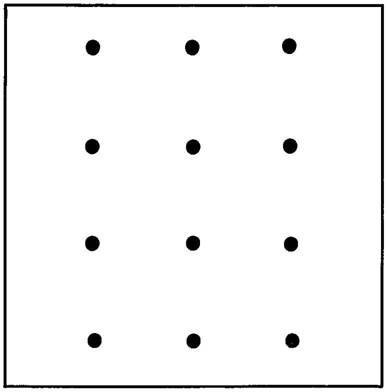
		8		4

Complete the grid and look for patterns.
Investigate for different sized boards.



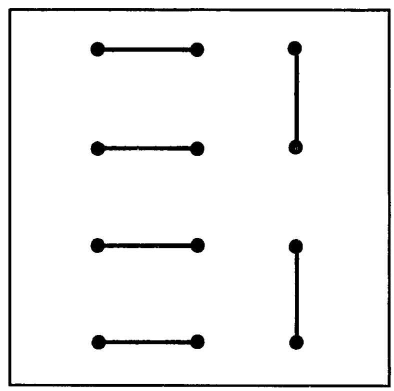
LINKS

Dotty paper

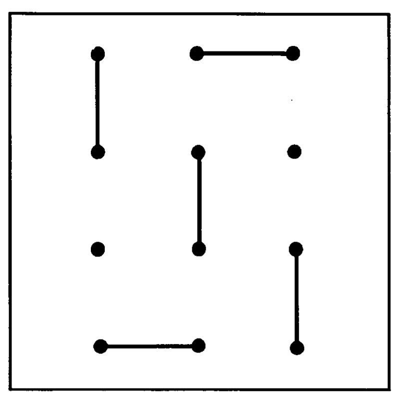


Start with a 4 × 3 grid of dots.
Join the dots in pairs.
No dot can be joined by more than one line.

Six lines, all dots joined.



Five lines, two dots unjoined.

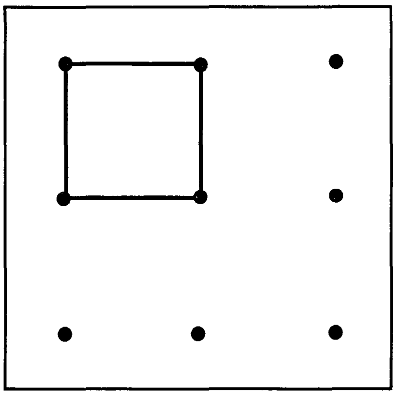


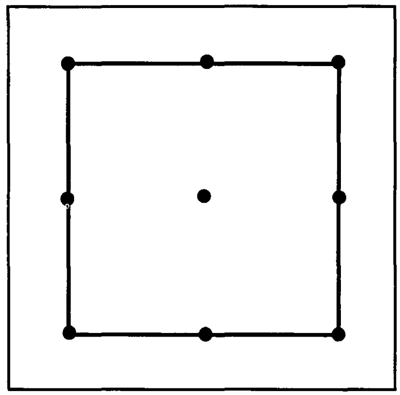
Investigate the number of possible unjoined dots.
Try using different sized grids.

GRID SQUARES

Geoboard
Dotty paper

Use a 3 × 3 grid on the geoboard.
Here are two different squares.

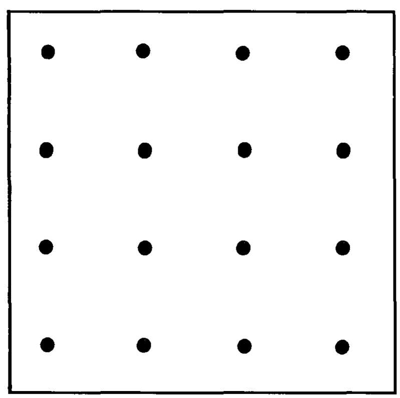




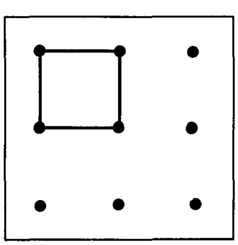
Can you find a third?

Investigate different squares on a 4 × 4 grid.

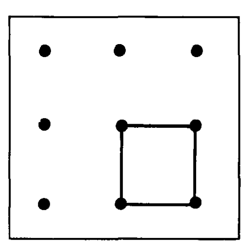
Record your results on dotty paper.



Try other sized grids.

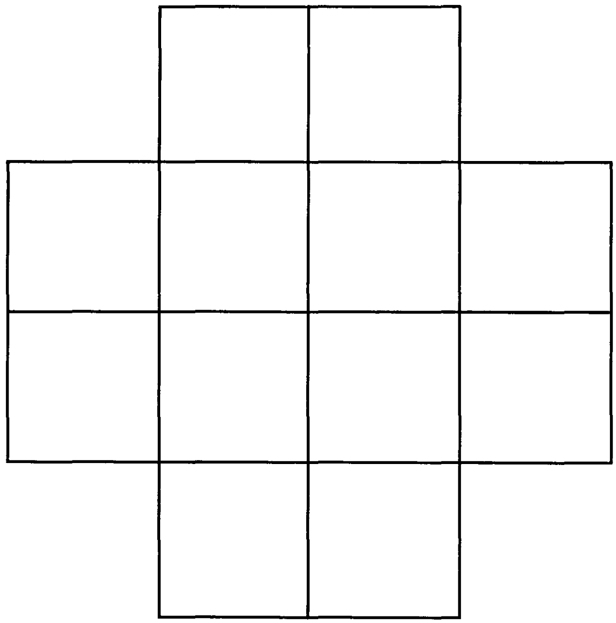


These squares are not different. They are the same size but in a different position.

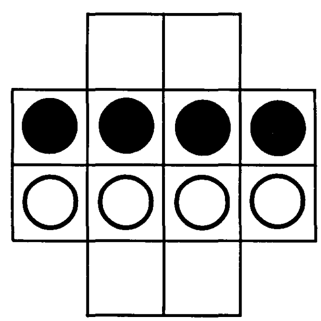


COLOUR EXCHANGE

Counters



Place four black and four white counters in this starting position.



A move consists of sliding a counter one space into an empty square, horizontally or vertically.
The aim is to reverse the position of the black and white counters.
How many moves are required?

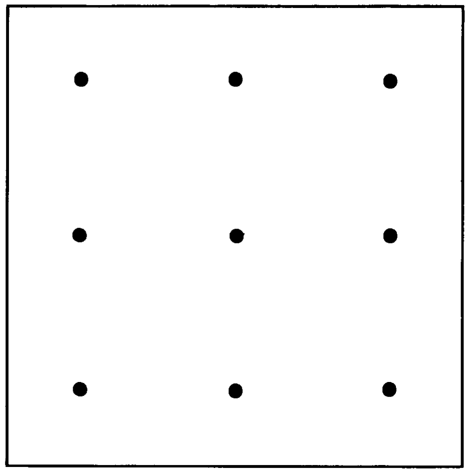
Investigate for different starting positions of the counters.

POLYGONS

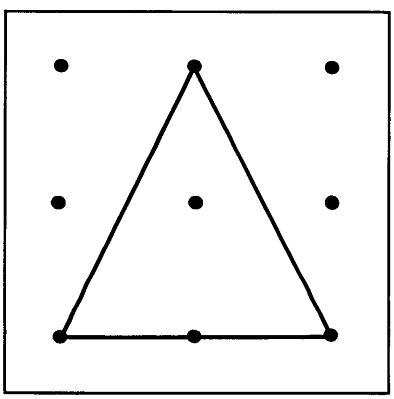
DOTS

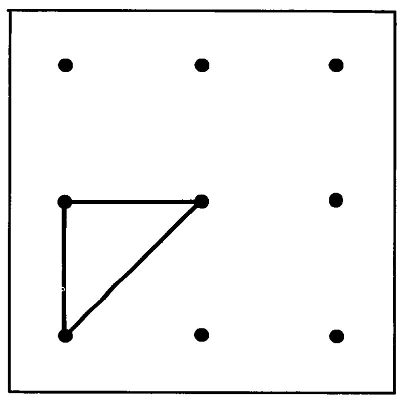
Dotty paper

Use a
3 × 3 grid.

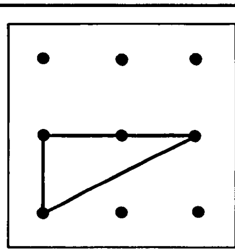


Here are some triangles made by using the nine dots:

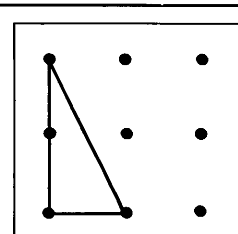




Can you find some more?
Investigate four-sided polygons.
Investigate the **areas** of your polygons.



These are the same.



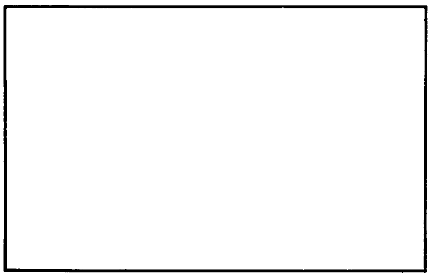
POLYGONS

PAPER CUTS

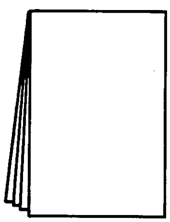
Scrap paper

Scissors

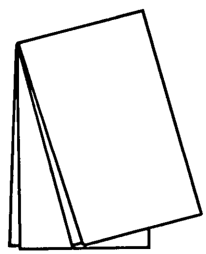
Take a piece of scrap paper. Fold it once.



Then fold again. There are two kinds of fold:

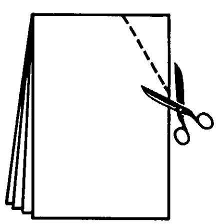


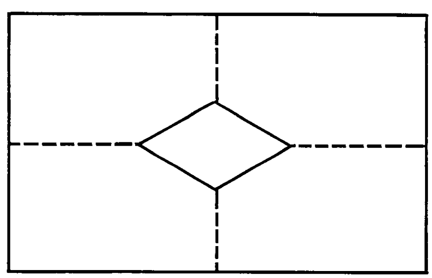
right-angled fold



oblique fold

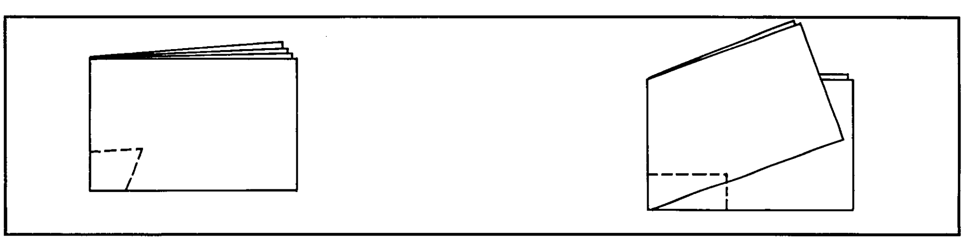
Cutting off the corner and opening the paper makes a shape.





rhombus hole

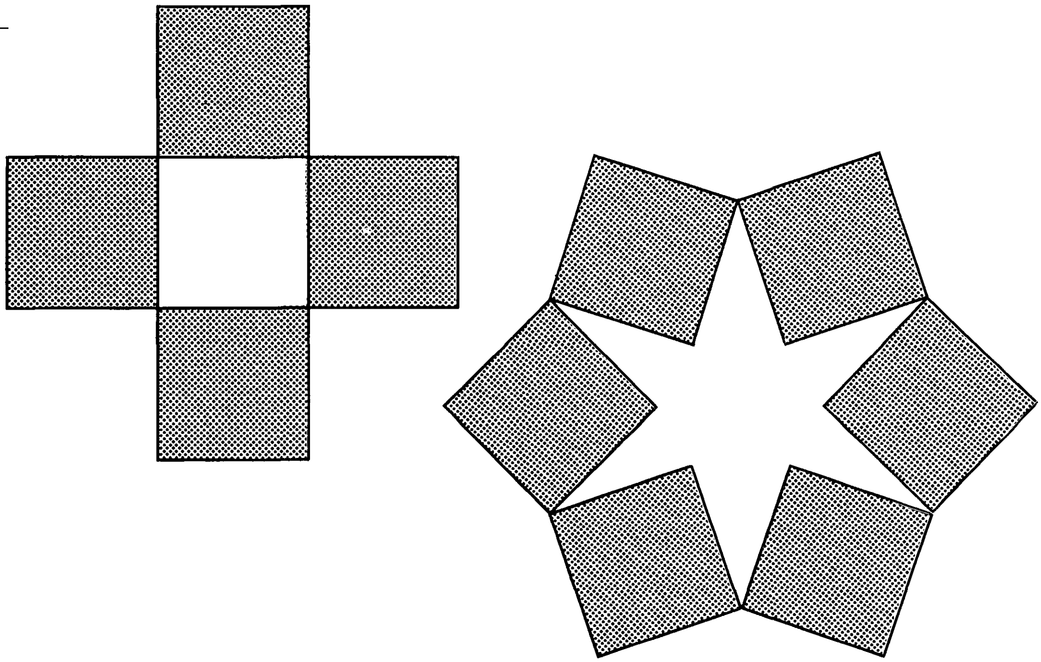
Investigate, trying different cuts on each type of fold.
Explore the shapes made if two cuts are allowed.



HOLES

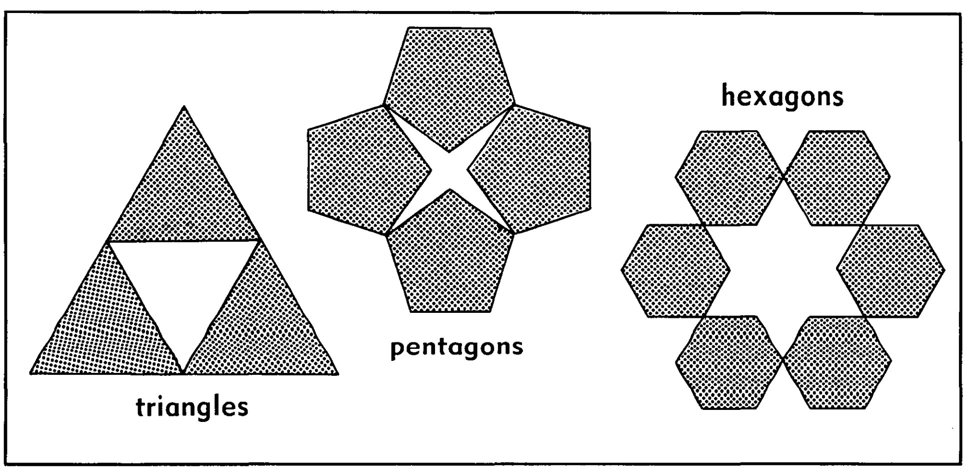
Regular polygons

Squares meeting at corners can leave holes.



Copy each of these by drawing round squares.
Make other holes using four and five squares.
Try using different numbers of squares.

Investigate holes made by other regular polygons.



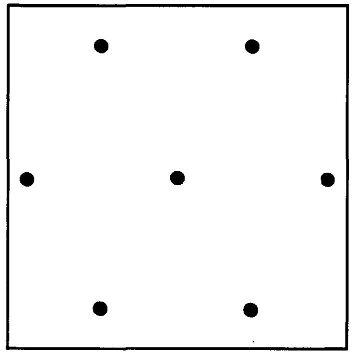
POLYGONS

20

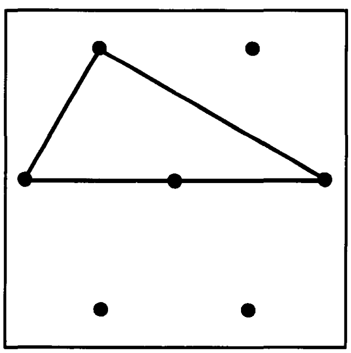
HEXA-DOTS

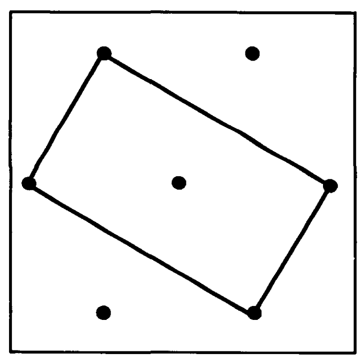
Hexagonal dotty paper

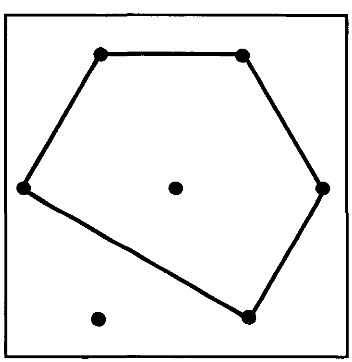
Use a seven-dot hexagonal grid.



Here are some polygons which can be made:







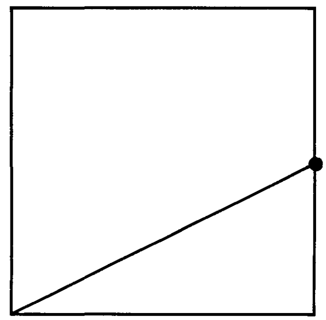
Can you find some more?

Investigate the **angles** of the polygons.

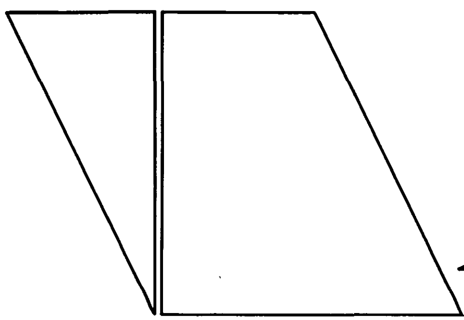
SQUARE CUTS

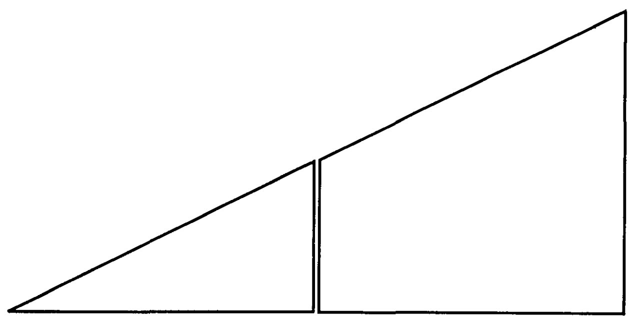
Card
Plain paper
Scissors

Draw a square of side 6 cm on card.
Mark the mid-point of one side.
Then draw a line as shown.



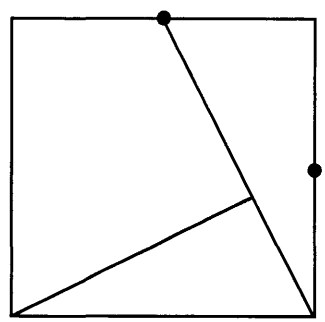
Cut out the square, then cut along the line to make two pieces.
By joining equal edges several polygons can be made:



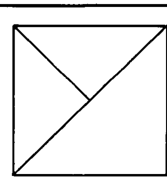


What other polygons can be made?

This square cut produces three pieces.
Investigate polygons made from the three pieces.



Invent some cuts of your own and investigate polygons.



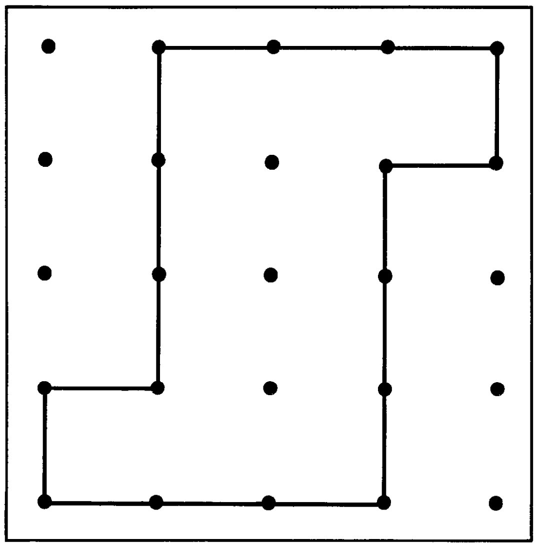
Record the polygons on plain paper.

POLYGONS 22

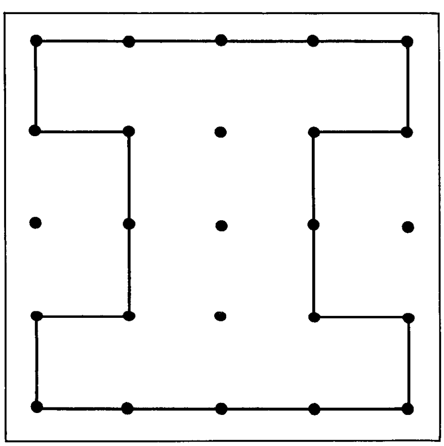
ROTATIONS

Dotty paper

Use a 5 × 5 grid.
This polygon has **rotational symmetry**.
It does not have **line symmetry**.



Investigate other polygons which have only rotational symmetry.



This polygon is not allowed.
It has rotational symmetry
and ***line symmetry.***

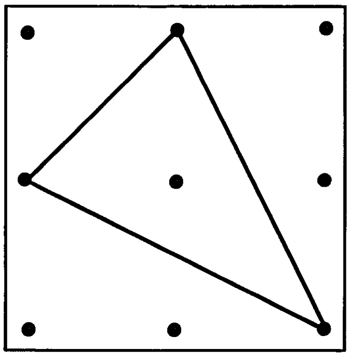
POLYGONS

SIDES

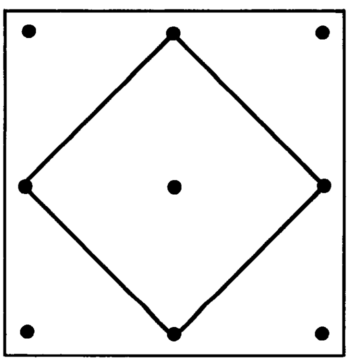
Geoboard

Dotty paper

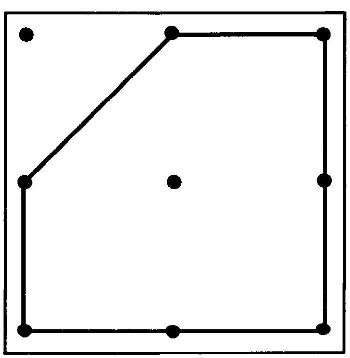
On a 3 × 3 grid we can make polygons with 3, 4, 5, 6 and 7 sides.



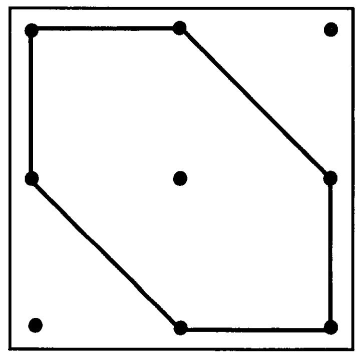
3 sides



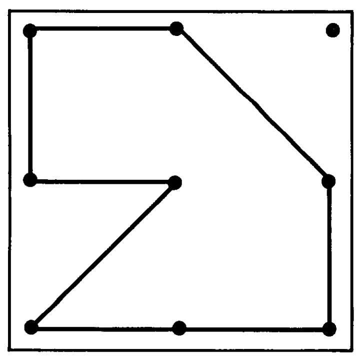
4 sides



5 sides



6 sides



7 sides

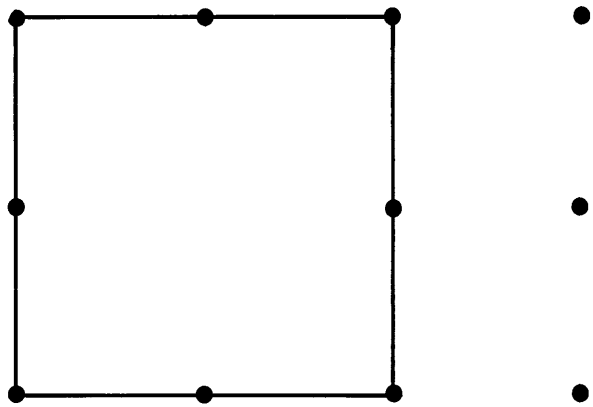
Now investigate polygons on a 4 × 4 grid.
Try using grids of different sizes and shape.

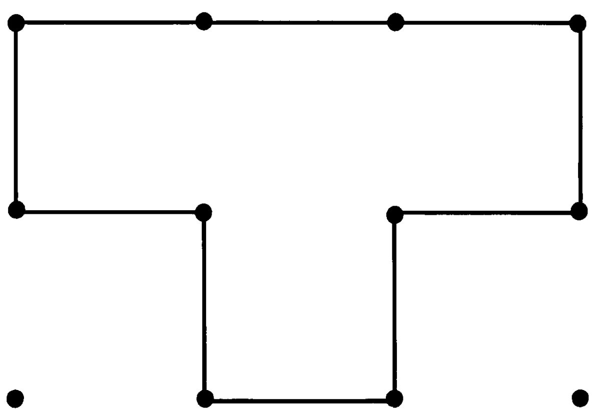
POLYGONS

AREAS

Dotty paper

Geoboards

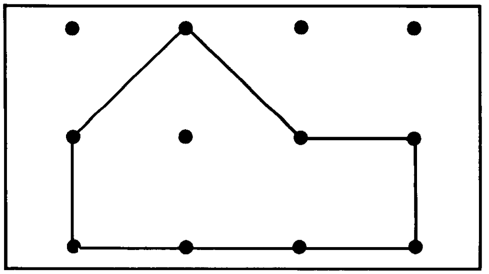




These polygons have an area of **four square units**. How many other polygons can you find which have an area of four square units?

Investigate polygons with areas of **six square units**.

Investigate polygons with other areas.



GAMES

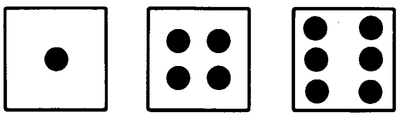
TRACK

Dice

Each player draws this grid.

1	2	3	4	5	6	7	8	9	10	11	12

Take turns to roll three dice.



Scores	
1 4 6	single scores
5 7 10	two-dice totals
11	three-dice total

Cross off the scores on the grid.

~~1~~	2	3	~~4~~	~~5~~	~~6~~	~~7~~	8	9	~~10~~	~~11~~	12

The winner is the first player to cross off all the numbers.

Play several games.

Change the rules so that numbers have to be crossed off in order.

You may not cross off 5, for example, until 4 has been scored.

~~1~~	~~2~~	~~3~~	~~4~~	5	6	7	8	9	10	11	12

Play to cross off 1 to 12 in order and then back again from 12 to 1.

~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	~~11~~	~~12~~

Investigate good opening throws.

What is the fewest number of throws possible to win the game?

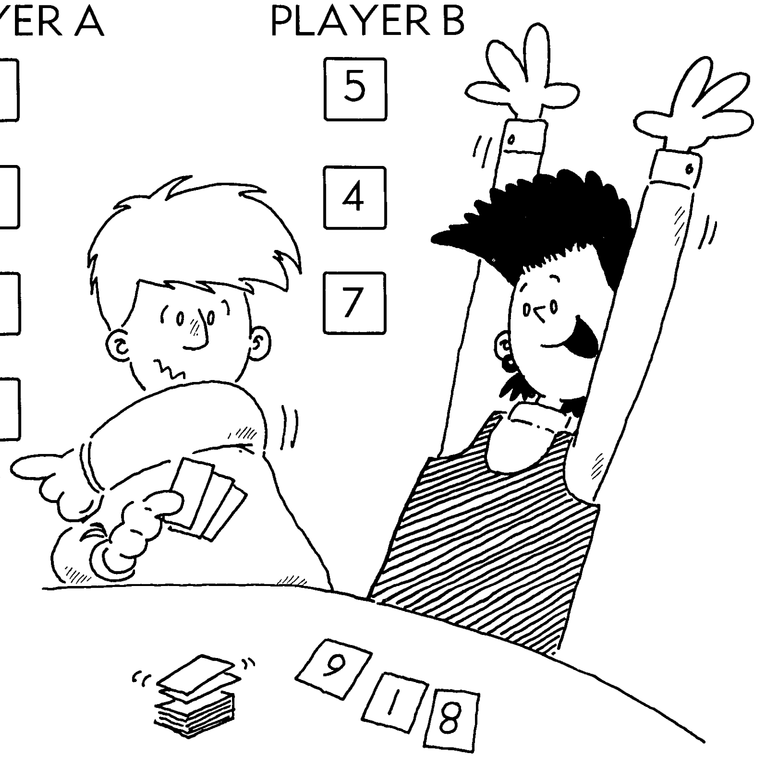
GAMES

FIFTEENS

Number cards 1 – 9

Place the cards face up on the table.
Players take turns to choose a card.
The winner is the first player to have a set of **three** cards which total 15.

	PLAYER A	PLAYER B
1st turn	1	5
2nd turn	2	4
3rd turn	6	7
4th turn	8	



Player A wins – 8
1
6 – total 15.

Play several games.
How many ways are there of scoring 15 with three cards?
Which cards make good opening choices?
Play the game with different totals.

GAMES

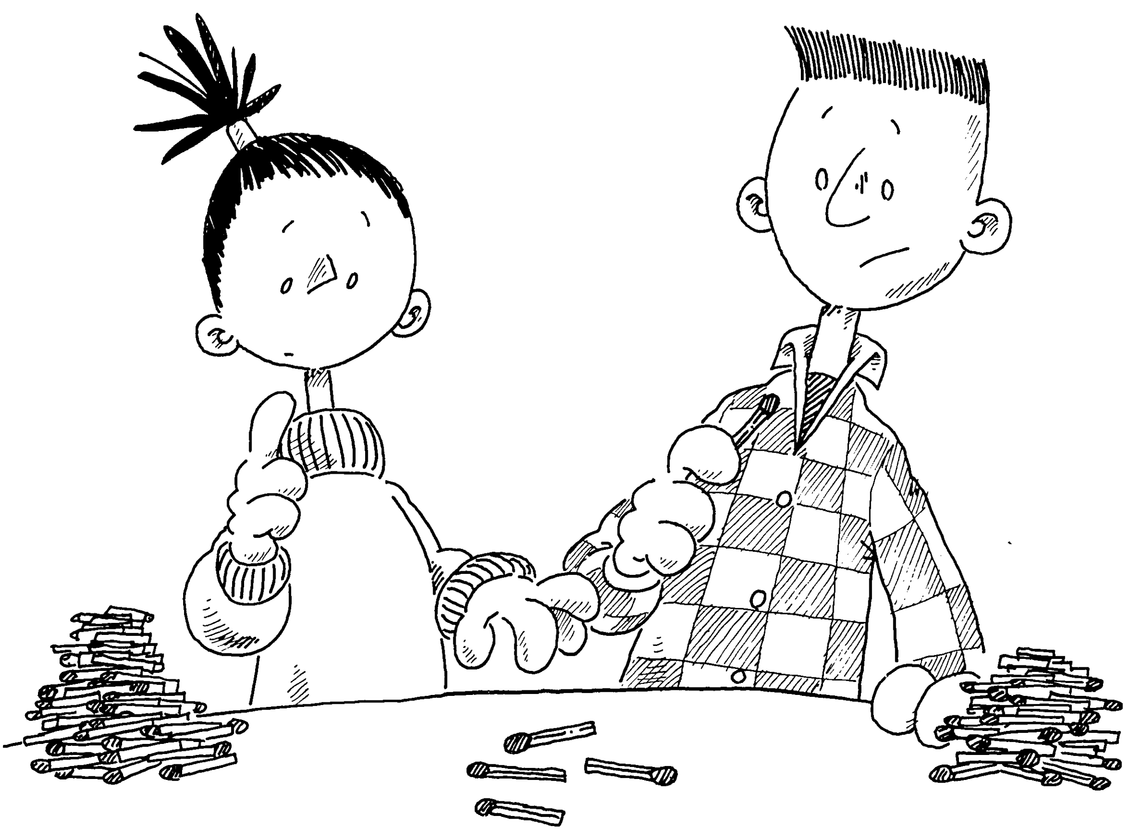
STICKS

Matchsticks

Start with 20 matchsticks.
Players take turns to remove one, two, three or four sticks.
The player who takes the last stick, or sticks, wins.

Play several games.
Work out a plan for winning.

Try starting with a different number of sticks.
What happens if players are allowed to take as many as six sticks?



TWO CARD SWOP

Number cards 1 – 25

Cards are chosen from the set 1 – 25.
One card is swapped for two.
They must have equal totals.

[18] *may be swapped for* [11] [7]	[10] *may be swapped for* [6] [4]

Place [25] in the centre of the table

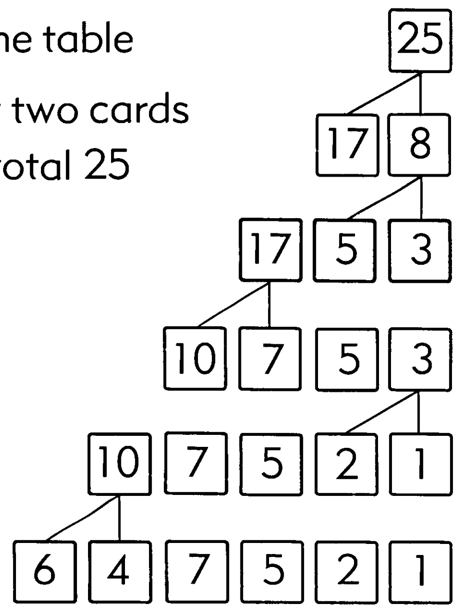
Player A swaps [25] for any two cards which total 25

Player B swaps [8]

A swaps [17]

B swaps [3]

A swaps [10]



Player B loses because no more swaps are possible.
Play several games.

The six cards left are [1] [2] [4] [5] [6] [7]

Investigate other possibilities.
Is it possible to have five cards left?

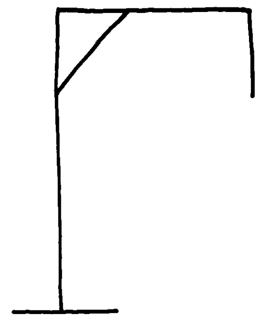
GAMES

13

HANGMAN

Calculator

Start with a scaffold.
Player A invents an equation
to fit these **seven** spaces.



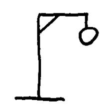
_ _ _ _ = _ _ _

Player B guesses digits (0 to 9) or signs (+, −, ×, ÷).

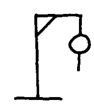
GUESS

1 YES _ _ 1 _ = 1 _ _

5 NO

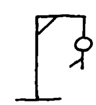


6 NO



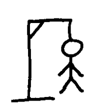
4 YES _ _ 1 _ = 1 4 4

+ NO



8 YES 8 _ 1 8 = 1 4 4

× YES 8 × 1 8 = 1 4 4 Player B wins.

Six wrong guesses lead to:  Then Player A wins.

Investigate different possible solutions to _ _ _ _ = 1 4 4

Play with **eight** spaces. _ _ _ _ _ = _ _ _

GAMES

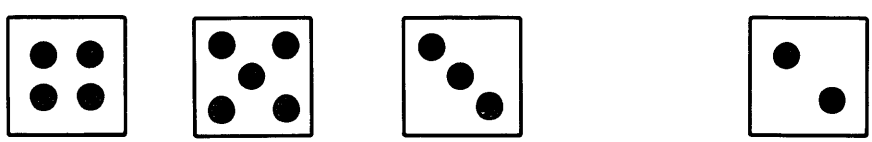
LOW SCORE

Dice

Throw four dice and arrange them in two groups.
Find the total of each group.
For each total score one point if it is:

ODD PRIME TRIANGULAR MULTIPLE OF 3

Scores four points
5 is **odd** and **prime**
9 is **odd** and a **multiple** of 3



Scores two points
12 is a **multiple** of 3
2 is **prime**

Have five throws each.
The player with the **lowest** score at the end wins.

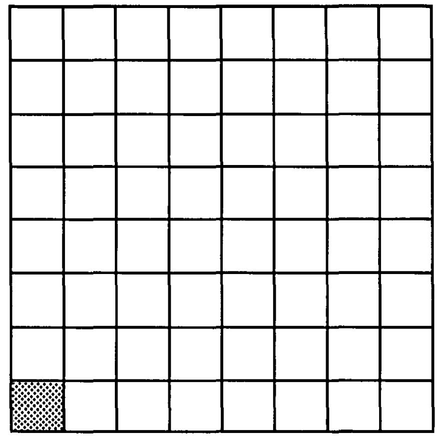
Which are the best totals to make?
Change the game to use **even** and **square** numbers.
Invent your own rules.

ATTRACTION

Squared paper

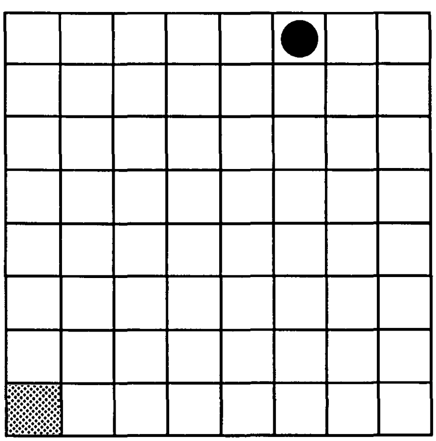
Counter

A game for two players



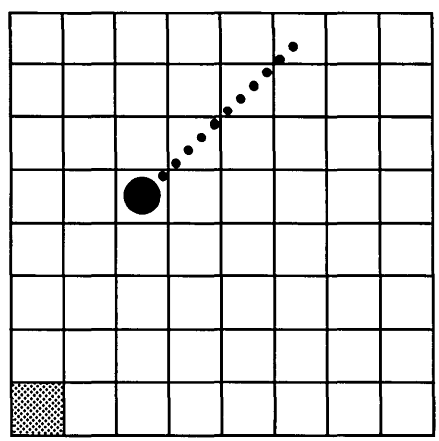
Draw an 8 × 8 grid.

One player places a counter on any square along the top row or the right-hand column.



The next player slides the counter **any** number of spaces in one of these directions:

↓, ←, ↙.



Players move alternately.

The player who moves the counter to the bottom left-hand corner square is the winner.

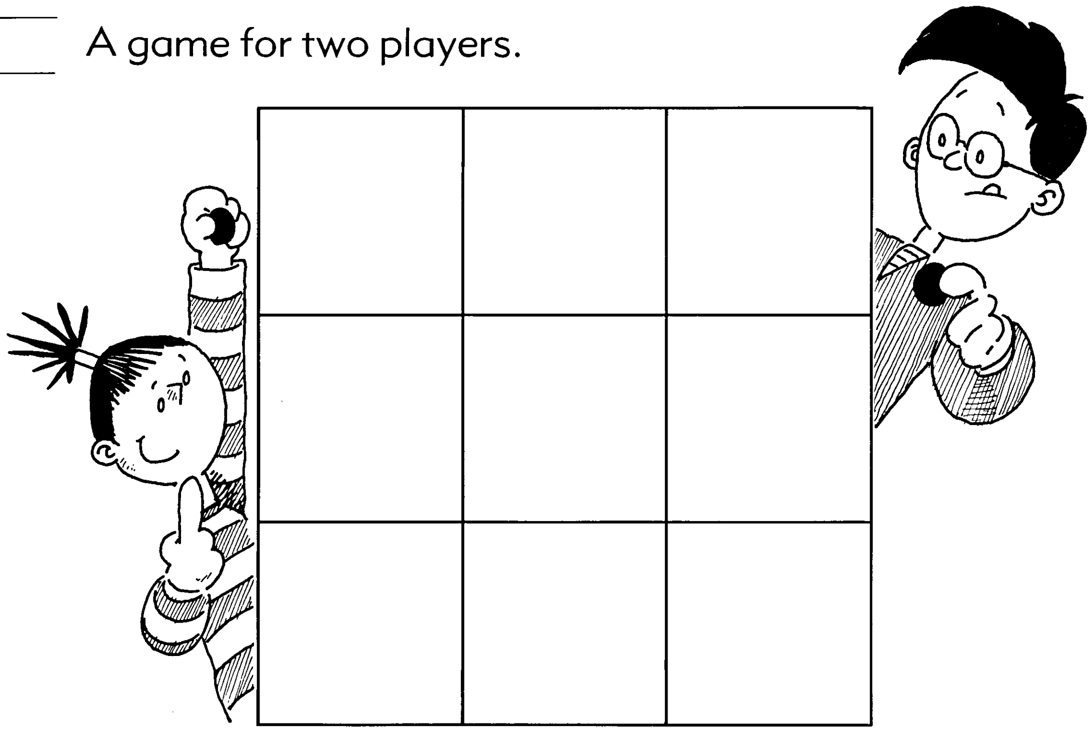
Play several games.

Can you find a way of winning?

GRIDDLE

Counters

A game for two players.



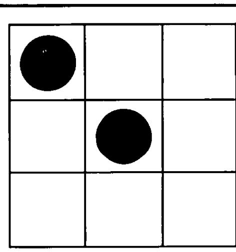
Take turns to place one, two or three counters on the griddle.

If two or three counters are placed, then they must be in the same row or column.

The player who places the last counter, or counters, wins.

Play several games.

Can you find a way of winning?



These counters are not in the same row or column.

NUMBER I

VERY ODD

Investigate sums of two consecutive odd numbers.

5 and 7 are consecutive odd numbers.

5 + 7 = 12

13 and 15 are consecutive odd numbers.

13 + 15 = 28

Try with three and four consecutive odd numbers.

5 + 7 + 9 = 21
13 + 15 + 17 + 19 = 64

NUMBER 1

FINAL DIGITS

These are multiples of 4.

4 8 12 16 20 24 28 32 36 ...

The final digits are

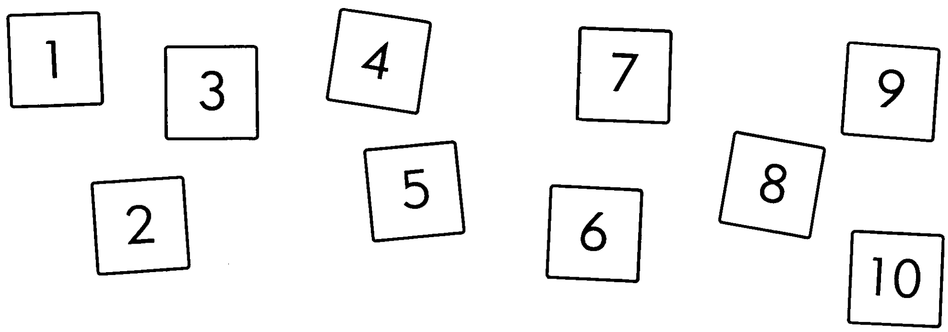
4 8 2 6 0 4 8 2 6 ...

What do you notice about the final digits?

Investigate the final digits of multiples of 6.
Investigate patterns for other multiples.

RIPE PAIRS

Number cards 1 – 10



Place the cards in pairs so that each total is a **square number**.

Here is a start:

4 5 total 9

9 7 total 16

How many pairs are possible?

Investigate other square number pairs.
Invent different rules for pairing the cards.

odd number pairs   ***prime number pairs***

POSITIONS

1 cm squared paper

This is a four-column grid.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21			

Copy and continue the sequence of numbers.

Investigate the position of some special numbers on the grid.

***Try:** odd numbers*
even numbers
multiples of five
prime numbers.

This is a six-column grid.

1	2	3	4	5	6
7	8	9	10	11	12
13	14				

Continue this sequence.

What are the positions of the special numbers now?

Investigate other grids.

SQUARE SUMS

Here are some square numbers:

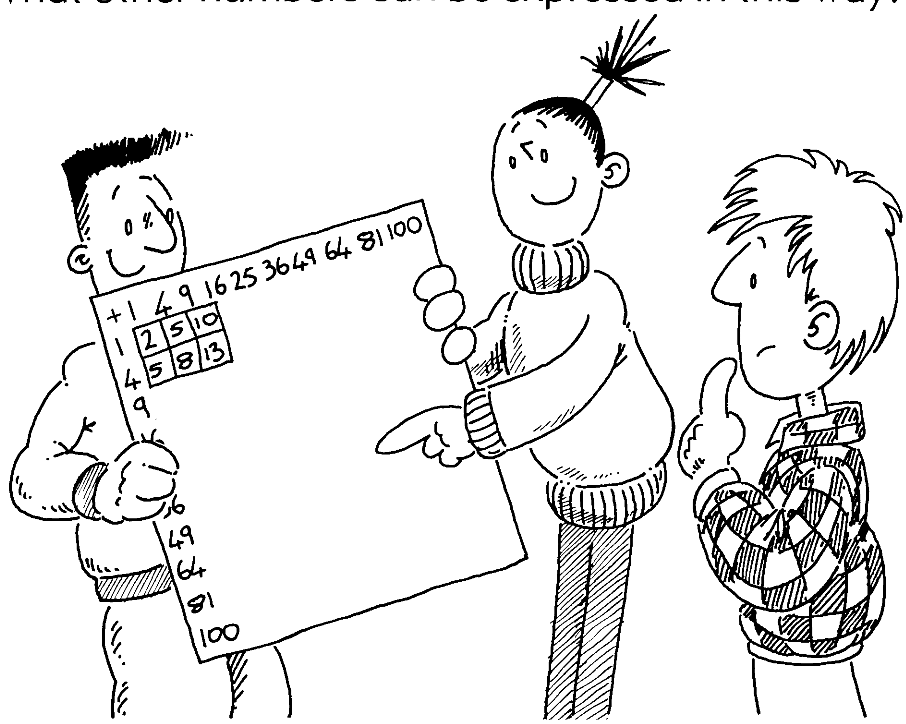
1 9 16 25 81 49 4 121 100 36 144 64

10 and 29 can be expressed as the sum of two square numbers.

$$10 = 1 + 9$$

$$29 = 25 + 4$$

What other numbers can be expressed in this way?



ISLAND-SPIRALS

1 cm squared paper

Copy this number spiral and continue it to 100.

	10	11	12	13
	9	2	3	14
	8	1	4	15
↑	7	6	5	16
21	20	19	18	17

Colour the **square numbers**.
Describe the position of the square numbers.

Copy these spirals and continue them to 100.

14	15	16	17	18	19
13	2	3	4	5	20
12	1			6	↓
11	10	9	8	7	

13	14	15	16	17
12	1	2	3	18
11			4	↓
10			5	
9	8	7	6	

Describe the positions of the square numbers.
Invent some spirals of your own. Investigate the position of square numbers.

PRIME SUMS

Table of prime numbers

33 can be expressed as the sum of two prime numbers.

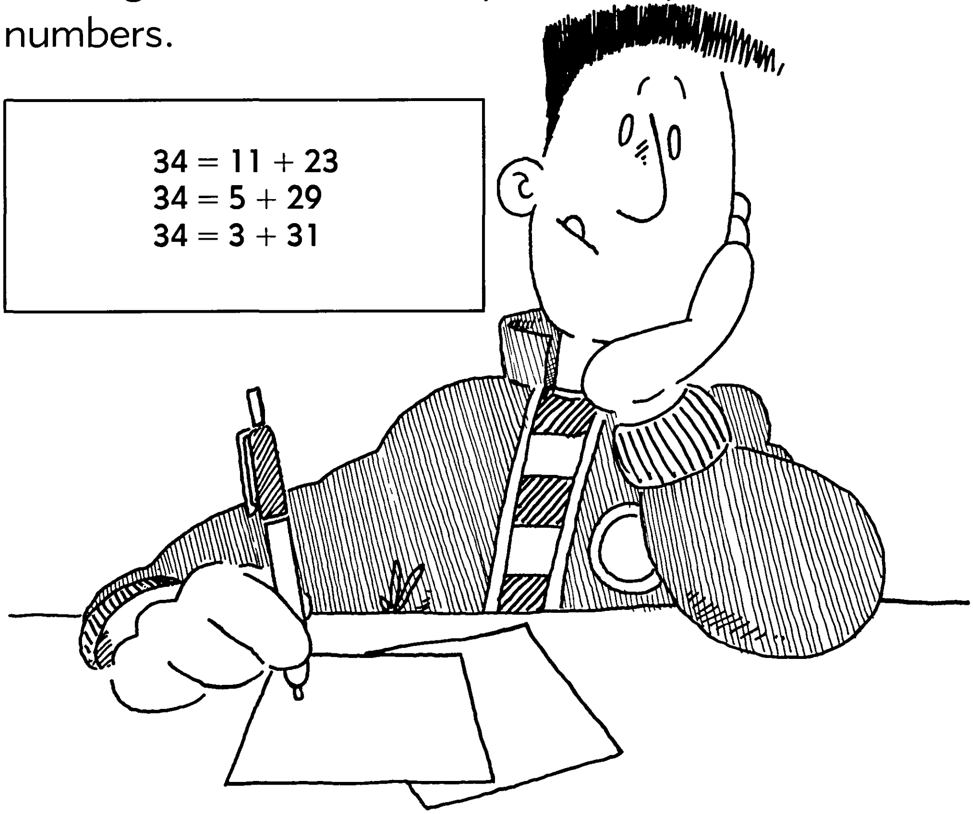
$$33 = 2 + 31$$

35 cannot be expressed as the sum of two prime numbers, but can be expressed as the sum of three primes.

$$35 = 5 + 13 + 17$$

Investigate the number of primes required for other numbers.

34 = 11 + 23
34 = 5 + 29
34 = 3 + 31



EXPRESS

Table of prime numbers

2 and 6 are **even numbers**.

They can be expressed as the **difference** between **consecutive prime numbers**.

6 = 29 − 23

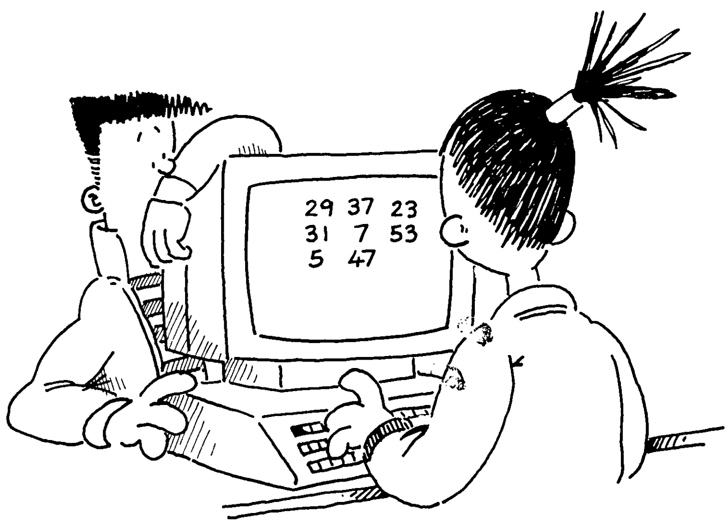
29 *and* 23 *are consecutive prime numbers.*

2 = 7 − 5

7 *and* 5 *are consecutive prime numbers.*

Investigate for other even numbers.

6 = 37 − 31
6 = 53 − 47

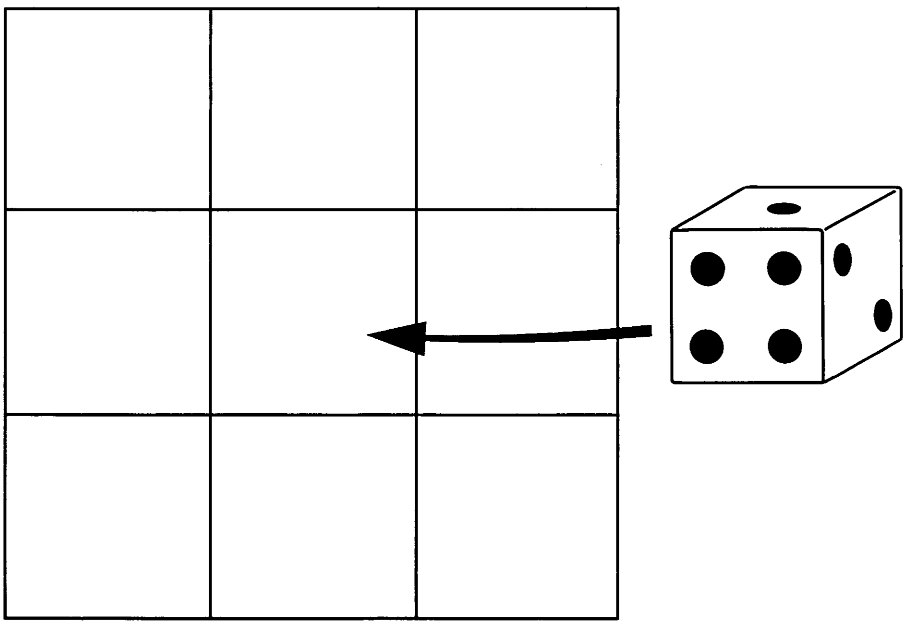


CUBES

DICE ROLL

Dice

Measure the sides of the dice.
Draw a 3 × 3 grid so that the dice will fit in the squares.

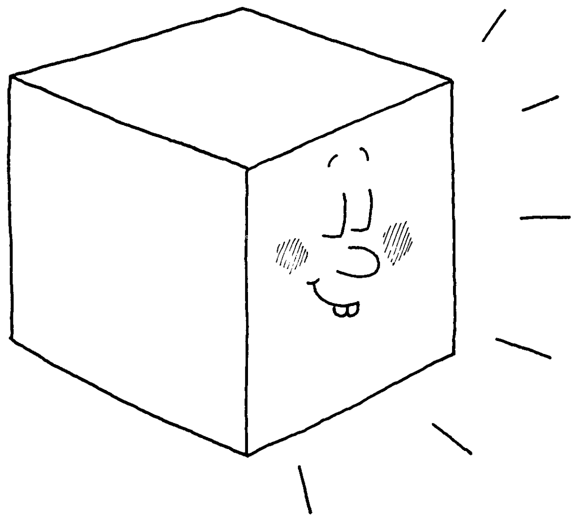


Always start on the centre square with the 1-spot on top and the 4-spots facing you.
Turn the dice over on one edge to reach other squares.
Which numbers can become the top number with two rolls?
Try three rolls.
How many rolls are needed to make six the top number?
Find ways of making one the top number in each of the other squares.
Try this for other numbers.

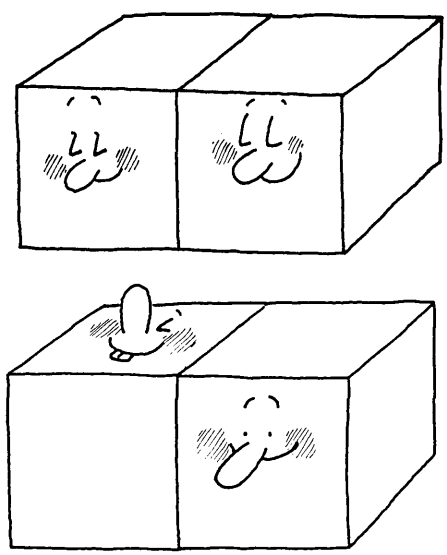
BLUSHING CUBES

Cubes

BLUSHING cubes have one red face.



You can join two of them face to face.
Here are two possible arrangements:



How many more can you find?
Investigate different arrangements with three blushing cubes.

CUBES

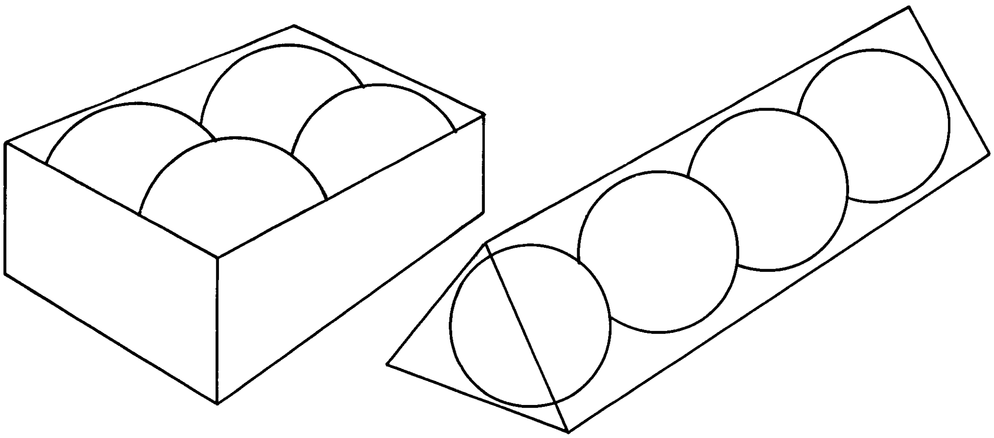
TIDY BOXES

Table-tennis balls

Card

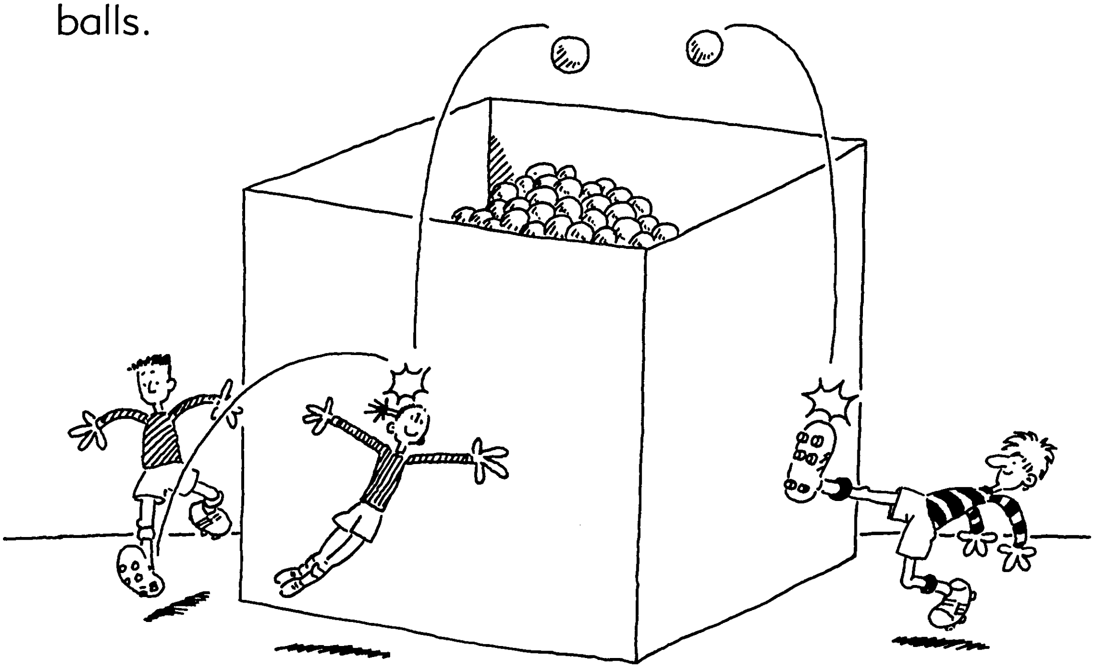
Sticky tape

Four table-tennis balls have been packed neatly into an open box.



Design and make some boxes to hold four table-tennis balls.

Try designing boxes to hold a different number of balls.

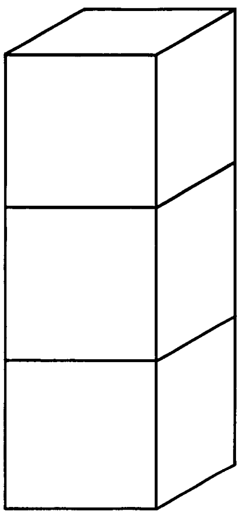


CUBES

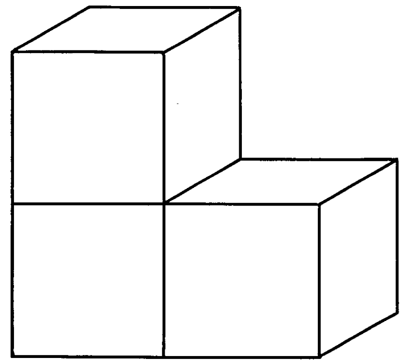
BLOCKS

Cubes

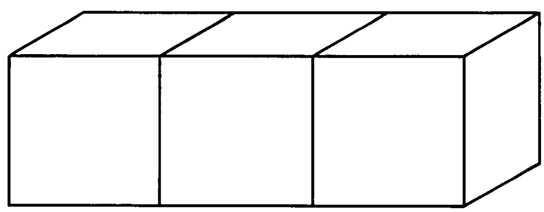
Join three cubes face to face to make these blocks:



three-storey block



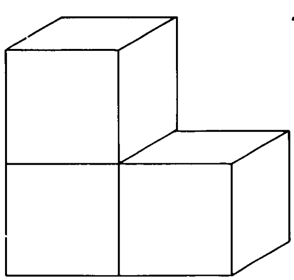
two-storey block

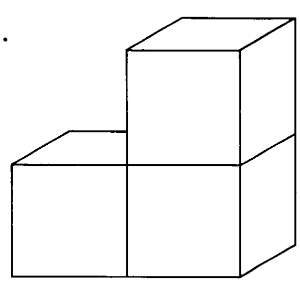


one-storey block

Can you find any more blocks?

These are the same.



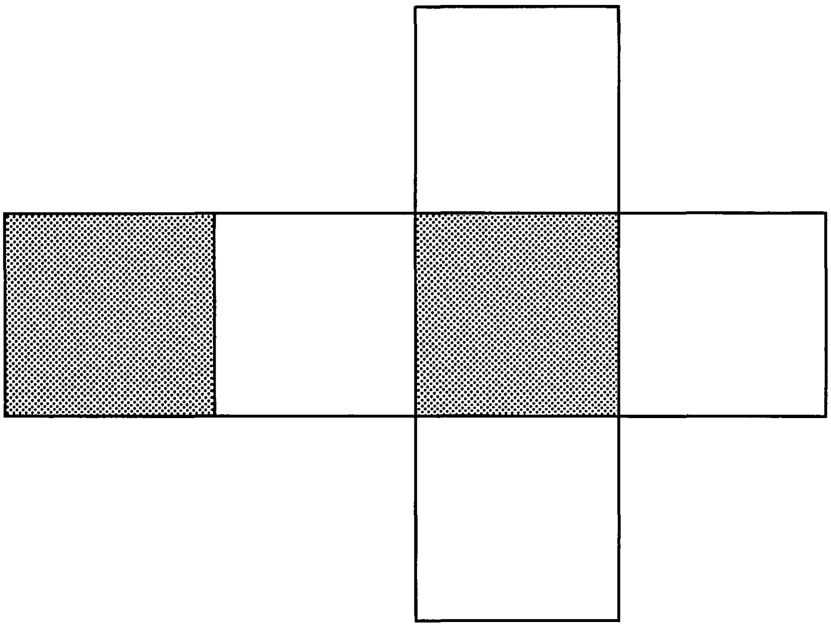


Investigate blocks built from four cubes.
Try five cubes.

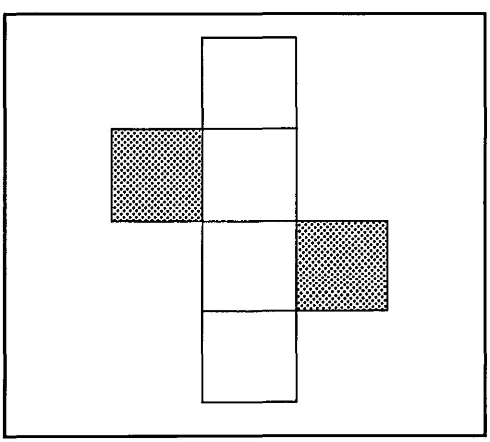
TWO-TONE

Squares
Squared paper
Sticky tape

Take four white and two red squares.
Join them to make the net of a cube.



When folded, the red squares must be **opposite** each other.
Investigate different ways of joining the white and red squares.

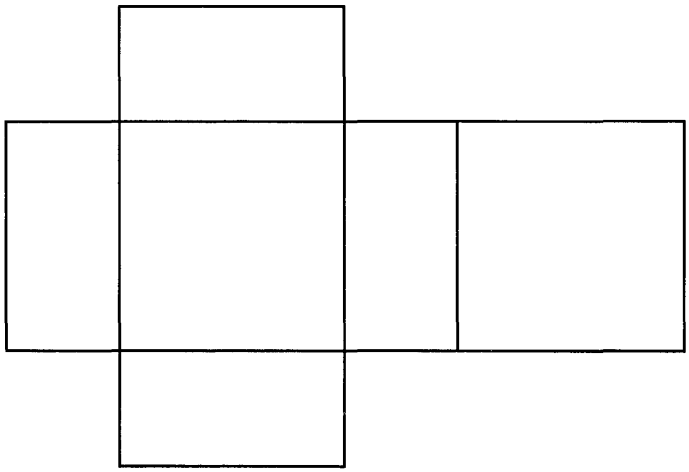


CUBES

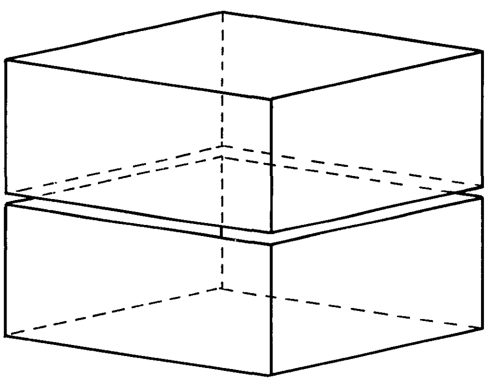
NET DESIGNS

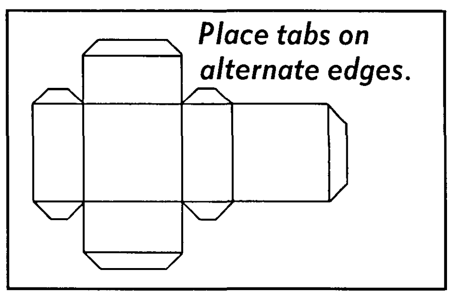
Card
Scissors
Glue

This net is designed to produce half a cube.

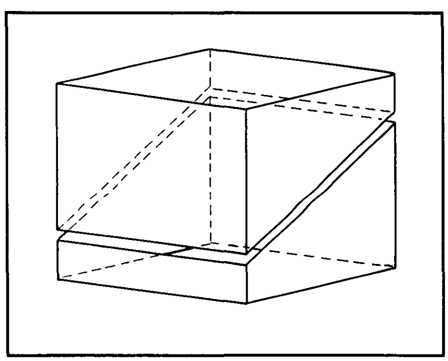


Draw two of the nets on card, add tabs, and make the models.





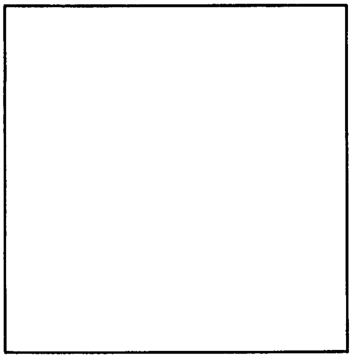
Design some more nets which produce half cubes.



Make models from your designs.

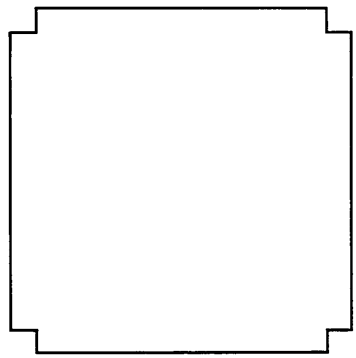
CORNER CUTS

Squared paper
Scissors
Calculator

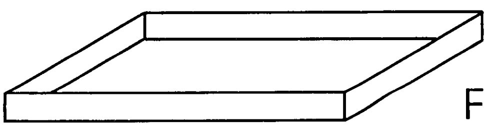


Cut out a 15 cm square.

Cut a 1 cm square from each corner.



Fold to make an open box:



Find the **volume** of the box.

Cut a 2 cm square from each corner and find the new volume.
Investigate what happens to the volume if different sized squares are cut from the corners.

Show your results in a table.

size of corner square cm	*size of box* cm	*volume* cm^3
1	$1 \times 13 \times 13$	169
2	$2 \times 11 \times 11$	242

FACES

Cubes

Take two cubes.
Write 1, 2, 3, 4, 5, 6 on the faces of one cube.
Write 3, 4, 5, 6, 7, 8 on the faces of the other.

| 4 | From above, one cube will show a **single-digit** number.

| 4 | | 7 | Two cubes can be placed to show a **two-digit** number.

Which single-digit and two-digit numbers can be shown?
How many **square numbers** can be shown?

Two other cubes have some **blank** faces.
One has 2, 3, 5, 6, 8 __ and the other has
1, 2, 4, __, __, 9
How many **square numbers** can be shown now?

Write numbers on the faces of two cubes to show **all** possible single-digit and two-digit square numbers.
Suppose some faces are blank. Can it be done now?

What happens if you want to show prime numbers instead of square numbers?
Investigate for other numbers.

CALCULATOR

CALCULATOR

SWITCH

Calculator

$$\begin{array}{r} 39 \\ \times\ 62 \\ \hline 2418 \end{array}$$

Switch the digits around.

$$\begin{array}{r} 93 \\ \times\ 26 \\ \hline 2418 \end{array}$$

$$\begin{array}{r} 46 \\ \times\ 32 \\ \hline 1472 \end{array}$$

Switch the digits around.

$$\begin{array}{r} 64 \\ \times\ 23 \\ \hline 1472 \end{array}$$

Find other pairs of two-digit numbers which also work like this.

$$\begin{array}{r} \mathbf{84} \\ \times\ \mathbf{12} \\ \hline \mathbf{1008} \end{array}$$

Switch

$$\begin{array}{r} \mathbf{48} \\ \times\ \mathbf{21} \\ \hline \mathbf{1008} \end{array}$$

TWO DIGITS

Calculator

[0] [1]

The only digit keys you can touch are 0 and 1 but you can touch any other key.

Choose a target number.
Try and reach the target number in the least number of key touches:

e.g. TARGET 99

nine key touches

[1] [0] [0] [0] [0] [√] [−] [1] [=]

eight key touches

[1] [0] [×] [1] [0] [−] [1] [=]

seven key touches

[1] [0] [×] [=] [−] [1] [=]

six key touches

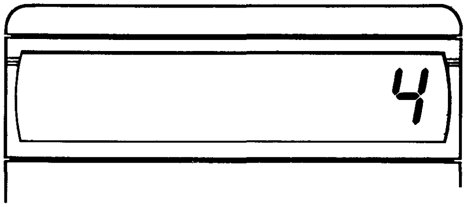
[1] [0] [0] [−] [1] [%]

Try different target numbers.

CALCULATOR

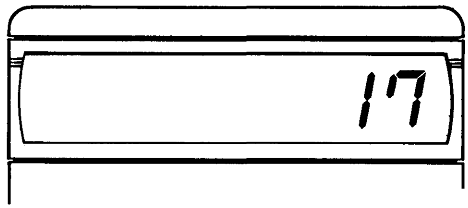
LIGHT BARS

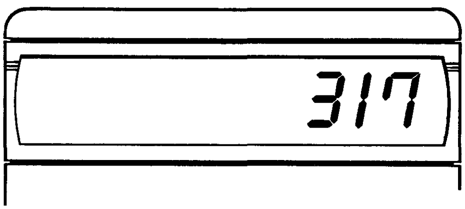
Calculator



The number 4 uses four light bars.

The number 17 uses six light bars.





The number 317 uses eleven light bars.

Which numbers can you make using seven light bars?

Investigate for different numbers of light bars.

68

FORBIDDEN KEYS

Calculator

Calculate: 78×36

[7] [8] [×] [3] [6] [=]

Suppose you are forbidden to use the [6] key.

Here are two ways of obtaining the answer:

[7] [8] [×] [3] [5] [+] [7] [8] [=]

[7] [8] [×] [7] [2] [÷] [2] [=]

[7] [×]

Find some other ways.
Choose other forbidden keys and investigate different ways of obtaining the answer.

Calculate: $1728 \div 36$

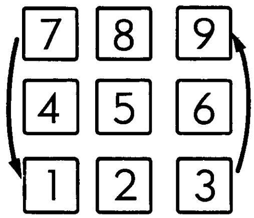
[1] [7] [2] [8] [÷] [3] [6] [=]

Choose a forbidden key.
Investigate different ways of obtaining the answer.

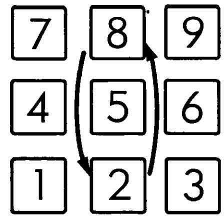
CALCULATOR

FINGER TAPPING

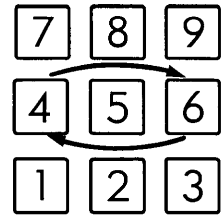
Calculator



71 + 39
= 110

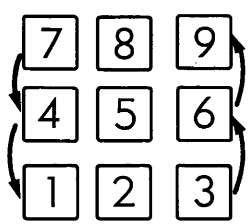


82 + 28
= 110

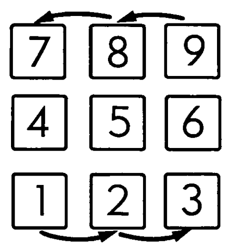


46 + 64
= 110

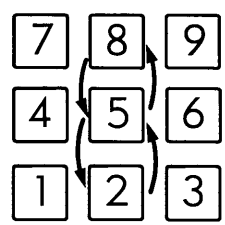
Find similar patterns producing totals of 110.



741 + 369
= 1110



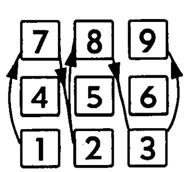
123 + 987
= 1110

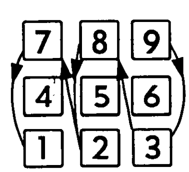


852 + 258
= 1110

Investigate further.
How many ways can you produce the totals 1110, 11110 etc?

Try some zig-zag patterns.

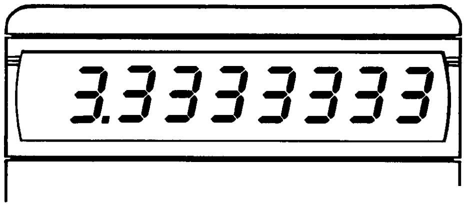




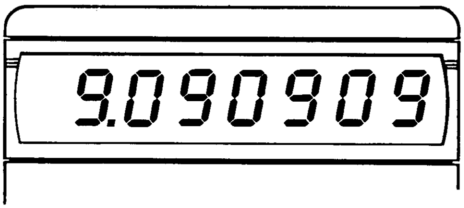
172839 + **938271** = **1111110**

CALCULATOR REPEATS

Calculator

10 ÷ 3 = 

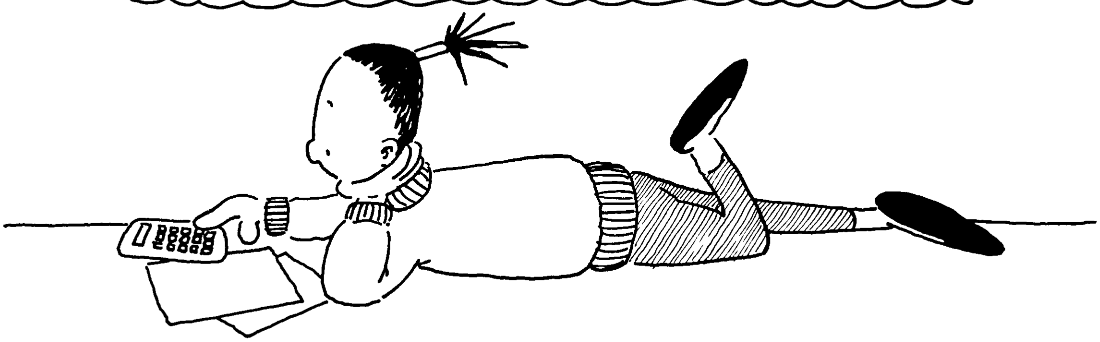
This is a single-digit repeat.

100 ÷ 11 = 

This is a two-digit repeat.

Investigate other divisions which produce:
a) single-digit repeats,
b) two-digit repeats,
c) three-digit repeats,

Rules: All answers must lie between 1 and 10.



CALCULATOR SOLITAIRE

Calculator

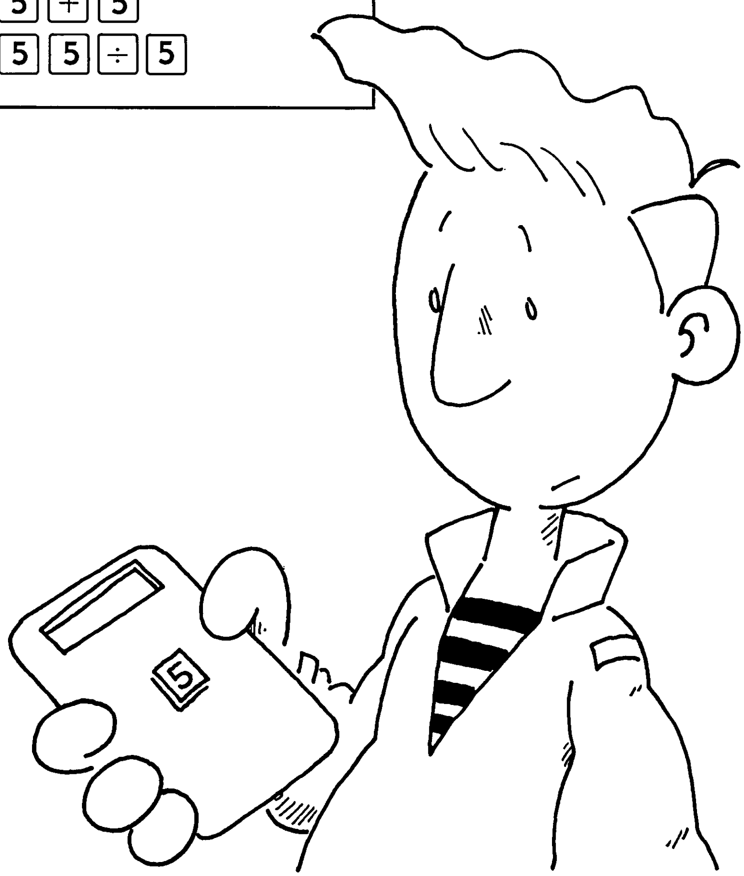
Only one digit key may be touched. [5]

It may be touched as often as you like.

Other non-digit keys may be used. [×] [√] [M+]

Investigate ways of obtaining answers in the range 1 to 20.

10 = [5] [+] [5]
11 = [5] [5] [÷] [5]



BIG TIMES

Calculator

Number cards 0–9

2	3
×	4
9	2

Use 2,3,4 and the multiplication sign.
Find the largest possible product.

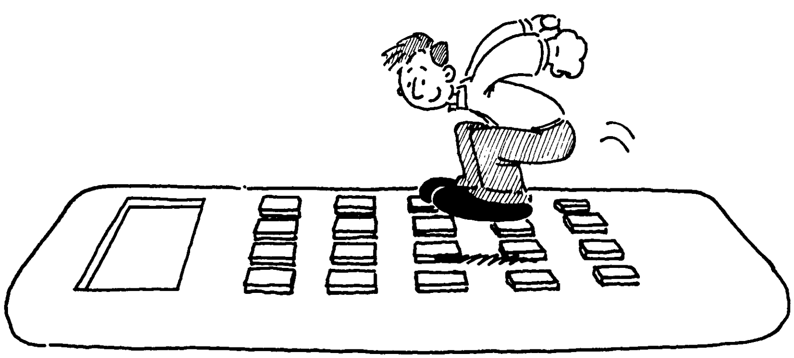
Investigate largest products with other sets of three digits.

Investigate largest products with four and five digits.

Write the digits on card.

3	4	5
	×	6

	3	4
×	5	6

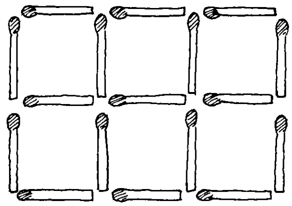


SQUARES

TAKE-AWAY

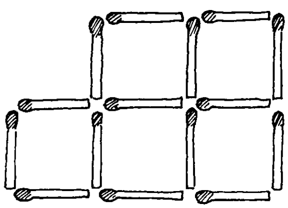
Matchsticks

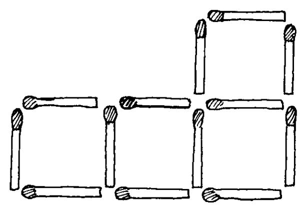
Here 17 sticks are used to make six squares:



Two sticks can be removed to leave five squares.

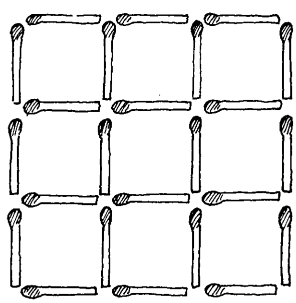
Five sticks can be removed to leave four squares.



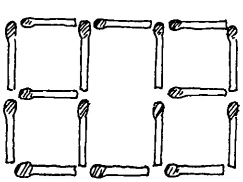


Investigate other possibilities.

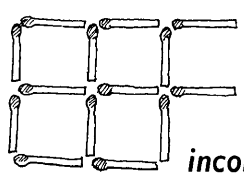
Try starting with 24 sticks to make nine squares.



These are not allowed:



not all squares



incomplete square

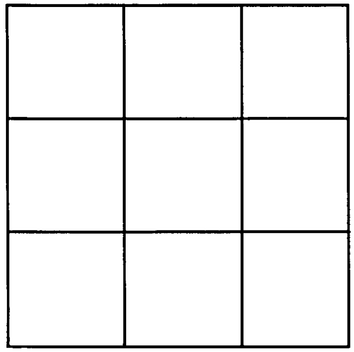
SQUARES

CUT-OUTS

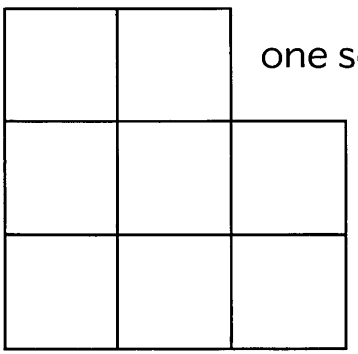
Squared paper

Scissors

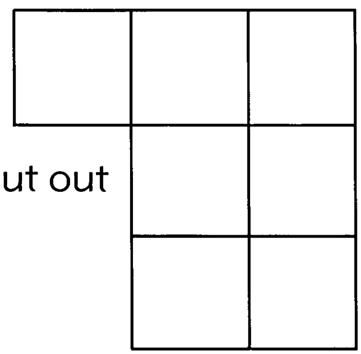
Start with a 3 × 3 grid.



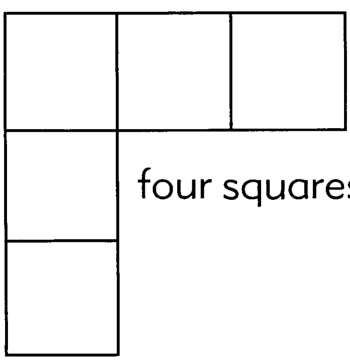
Shapes can be made by cutting out squares.



one square cut out



two squares cut out



four squares cut out

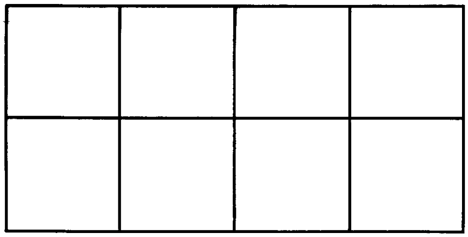
Investigate other shapes made by cutting out squares.

SQUARES

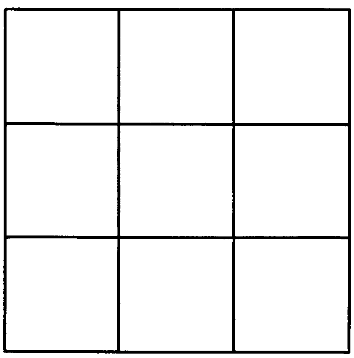
LINES

1 cm squared paper

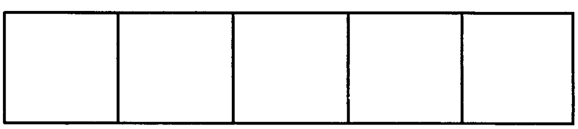
Each of the following three grids is made from eight straight lines.



There are 11 squares here. Can you find them?



Here are 14 squares.



Here are 5 squares.

Make different grids with seven straight lines.
How many squares are there in each grid?

Investigate grids with ten straight lines.

Results can be shown in a table:

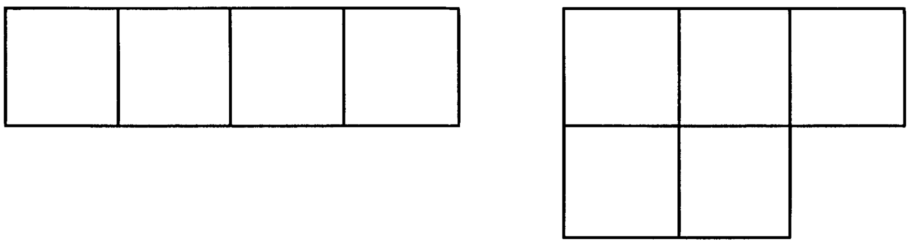
lines	squares
7	
8	5 11 14
10	

SQUARES

PERIMETERS

1 cm squared paper

These shapes have **perimeters** of ten units.

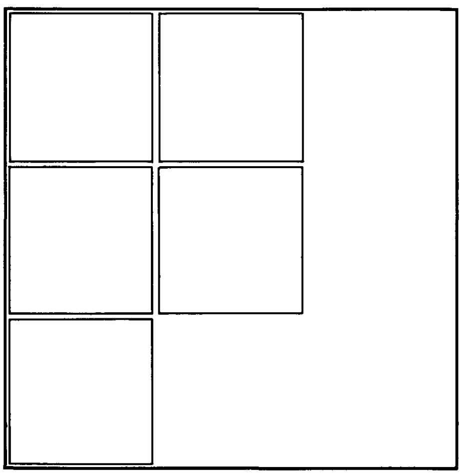


Can you find some more shapes with perimeters of ten units?
Investigate shapes which have a perimeter of 12 units.
Investigate shapes with other perimeters.

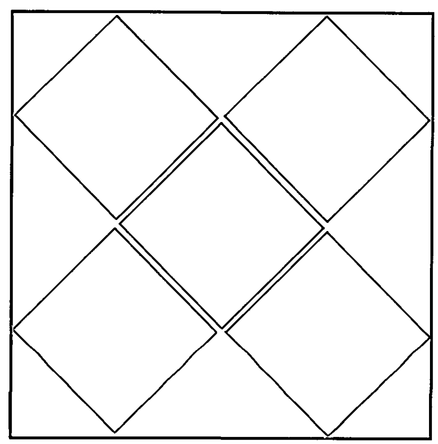
FRAMES

Squares

Five squares have to be arranged into a **square frame**.

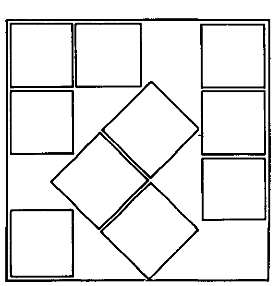


The squares must fit tightly with no overlapping.



How many different sized square frames can you find?

Try finding square frames for ten squares.
Investigate for different numbers of squares.



Ten squares in a square frame.

SQUARES 6

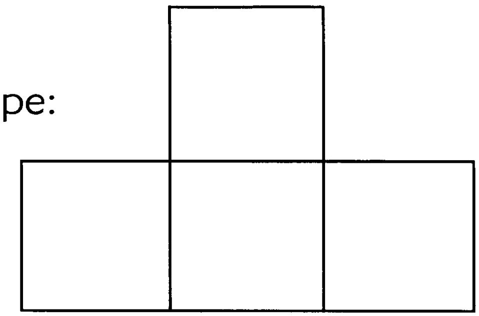
JOINING

Squares

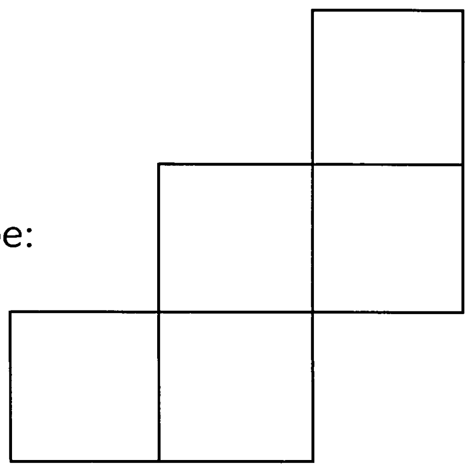
1 cm squared paper

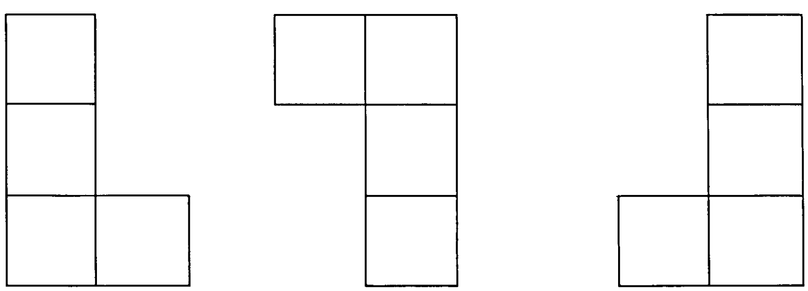
Investigate how many different shapes can be made by joining squares edge to edge.

Here is a four-square shape:



Here is a five-square shape:





These are not different.
They are the same shape but in a different position.

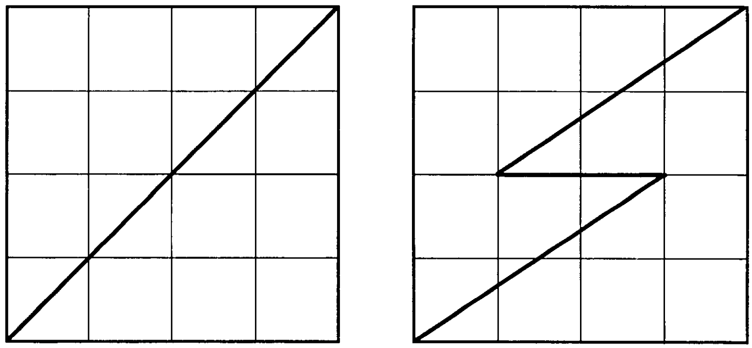
SQUARES

HALVING

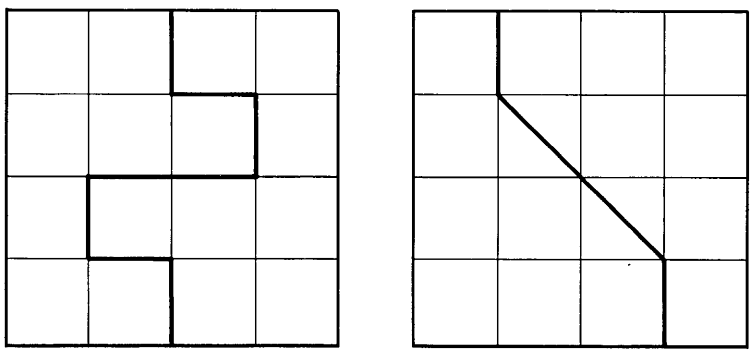
1 or 2 cm squared paper

Compasses

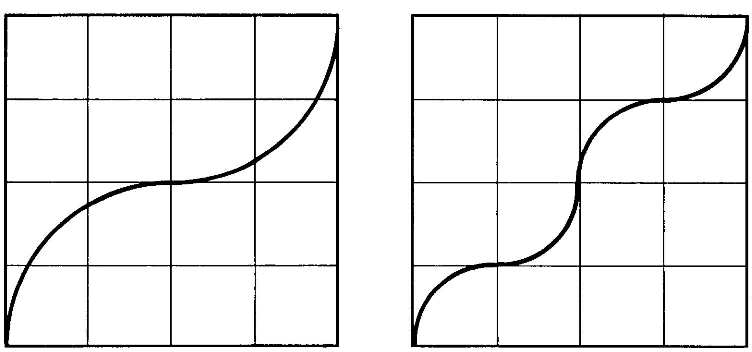
Here are some ways of halving a square:



straight lines from corner to corner



straight lines from side to side



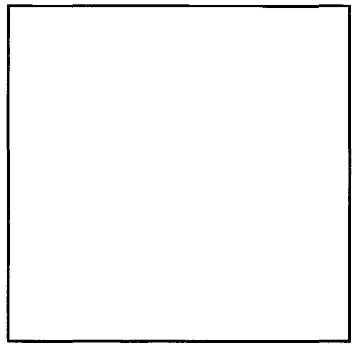
curved lines

Explore different ways of halving a square.

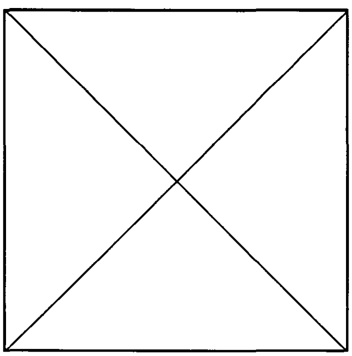
CHUNKS

Squares

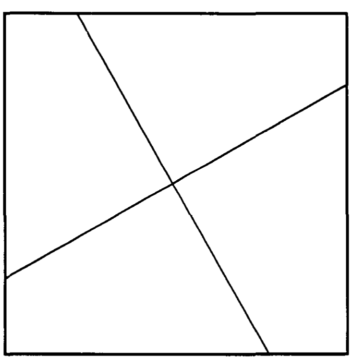
Start with a square.



You can draw two straight lines inside the square.



This produces four right-angled triangles.



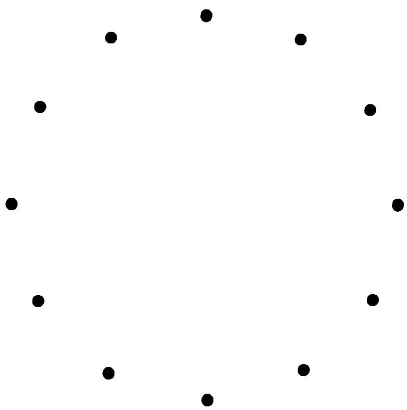
This produces four quadrilaterals.

Investigate other possibilites. Describe the shapes inside the square.
Try using three straight lines.

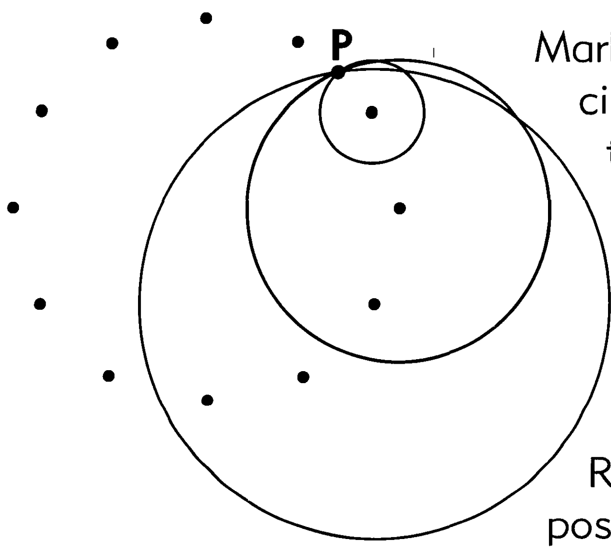
LOOPS

Circle paper

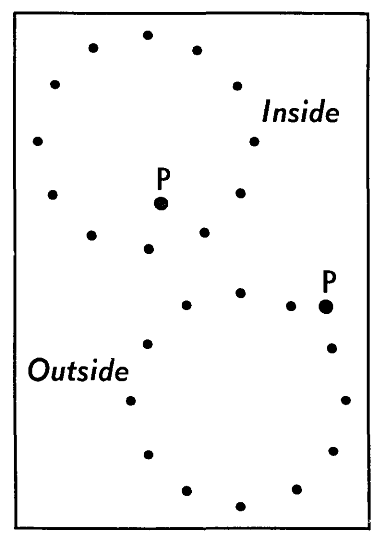
Compases



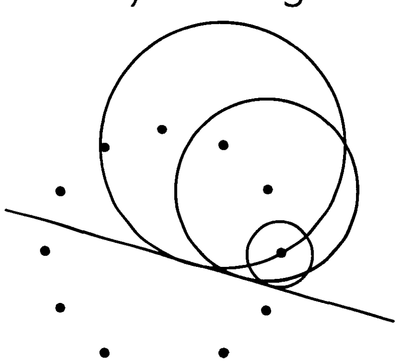
Use a circle of 12 equally spaced dots.

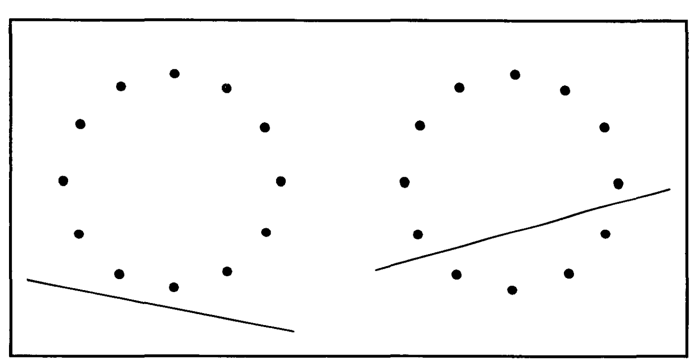


Mark a point P on the circumference. Place the compass point on each dot and draw circles. Each circle must pass **exactly** through point P. Repeat with different positions of P.



Try drawing the circles so that they touch a straight line.

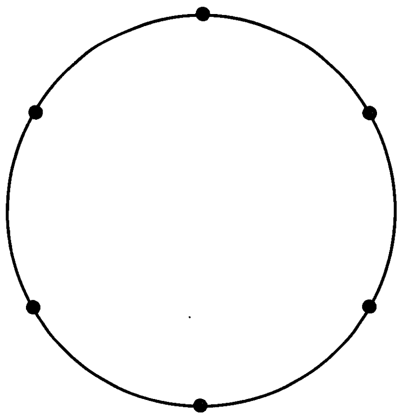




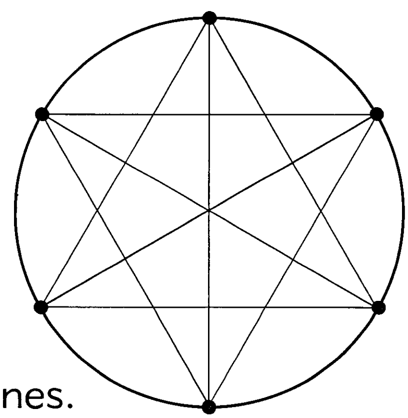
CIRCLES

DESIGNS

Circle paper

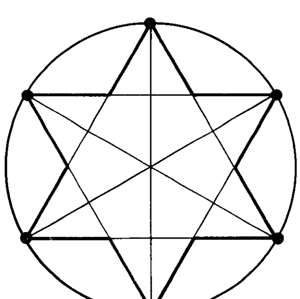


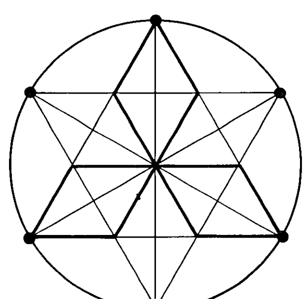
This circle has six equally spaced dots.

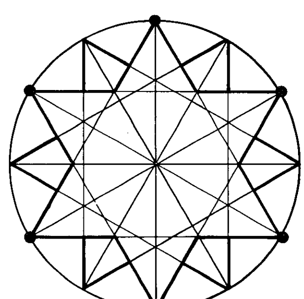


Join the dots with some faint lines.

Use the faint lines to create designs.







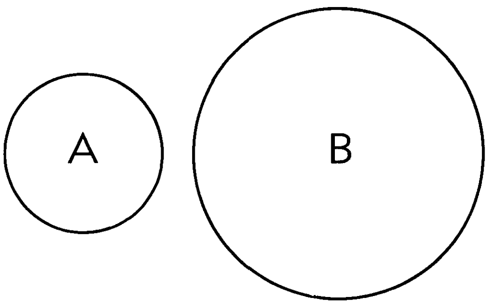
Create some designs of your own.
Draw extra guide lines if they are needed.

INS AND OUTS

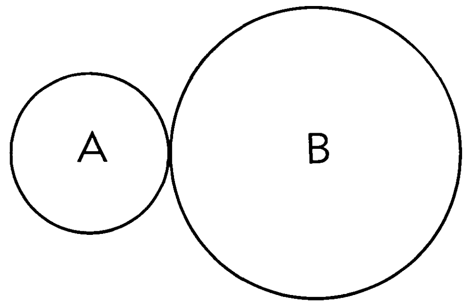
Compasses

Tracing paper

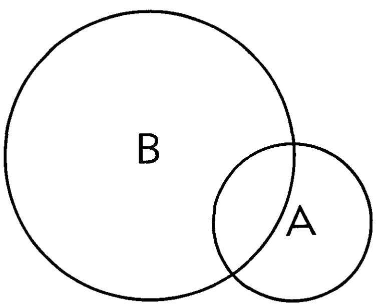
Two circles can be drawn in different positions.



AB outside

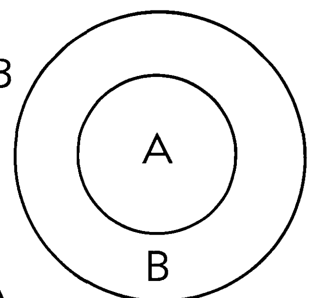


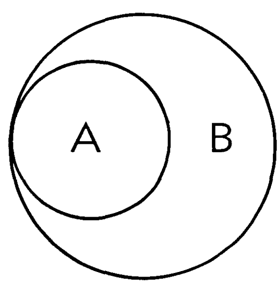
AB touching outside



AB cutting

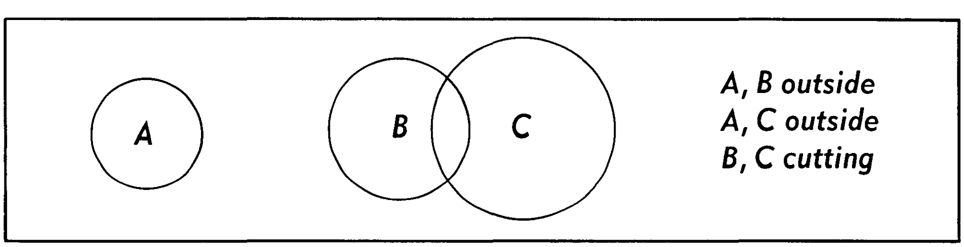
A inside B





AB touching inside

Investigate different positions for drawing three circles.



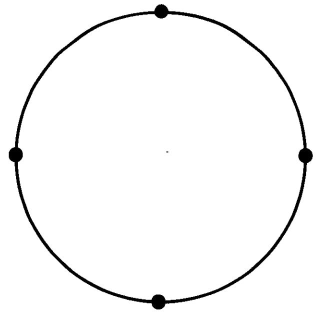
Label the circles and describe their positions.

CIRCLES

POLYGONS

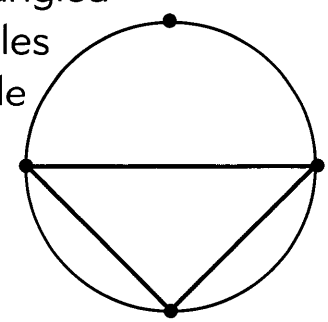
Circle paper

This circle has four equally spaced dots.

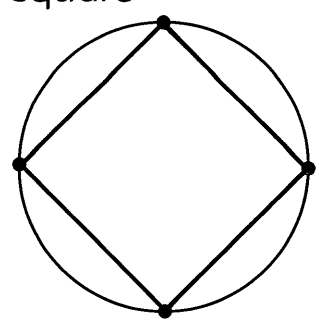


Here are two different polygons made by joining the dots:

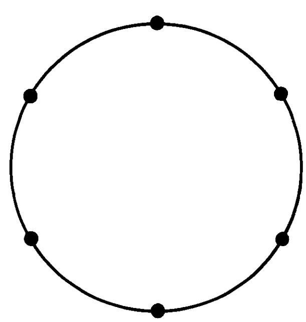
right-angled isosceles triangle



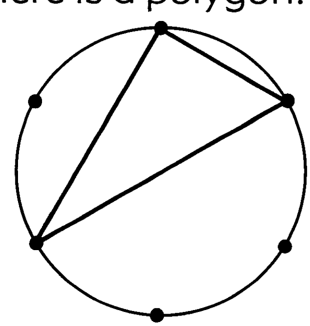
square



This circle has six equally spaced dots:



Here is a polygon:

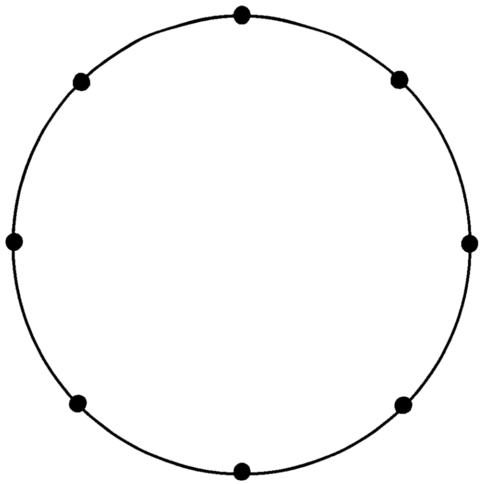


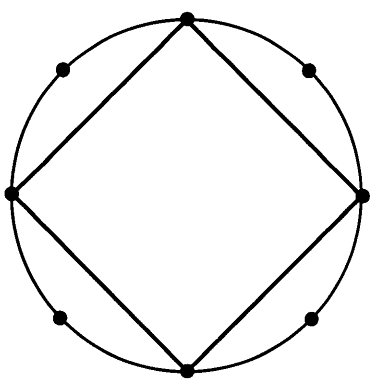
How many other polygons can you find?
Can you name them?
Investigate polygons on circles with different numbers of equally spaced dots.

PATTERNS

Circle paper

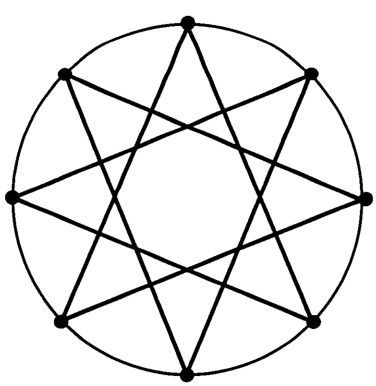
This circle has eight equally spaced dots.





Start at any dot and join every second dot.

This is a (8, 2) pattern.



Here every third dot is joined.

It is a (8, 3) pattern.

(8, 5)	(8, 1)

Investigate other patterns on eight equally spaced dots.

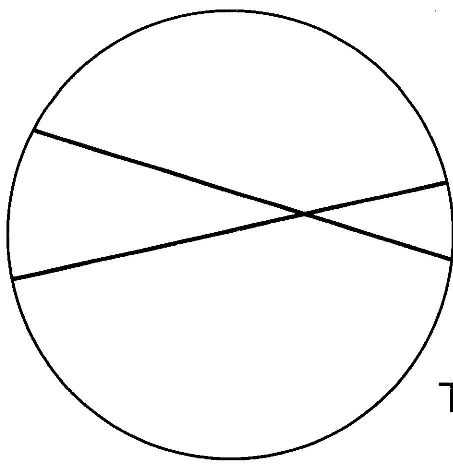
Try this for a different number of equally spaced dots.

CHORDS

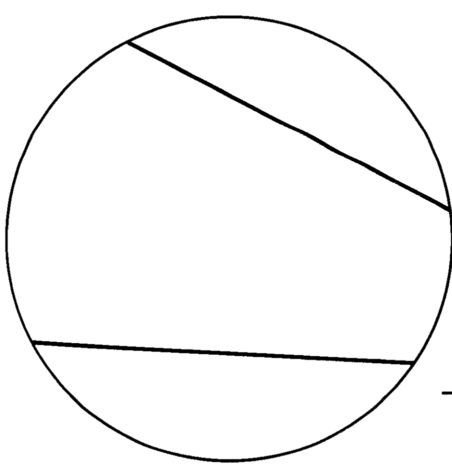
Compasses

Draw a circle.

Suppose you must draw two **chords.**



This produces four **regions**.



This produces three **regions**.

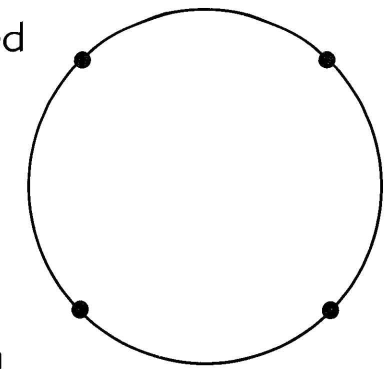
Try different ways of drawing three chords.

Investigate the number of regions produced.
Explore for other numbers of chords.

NO BREAKS

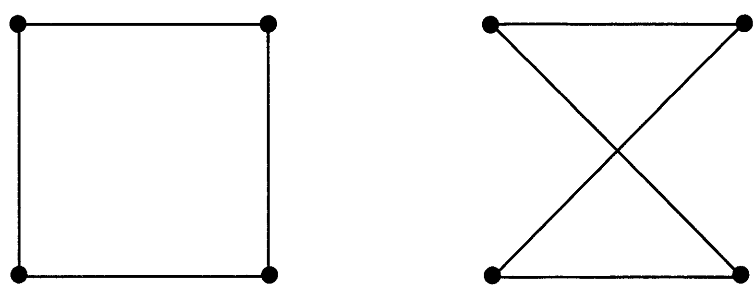
Circle paper

Here are four equally spaced points on a circle.

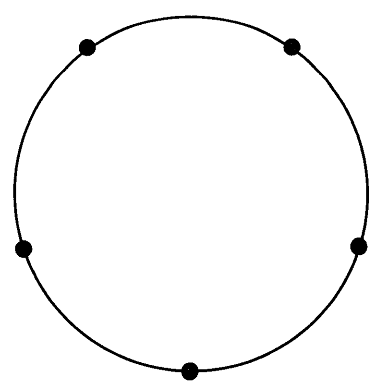


The points can be joined by straight lines without lifting the pencil off the paper, and passing through each point **once only**.

Two different paths are possible:

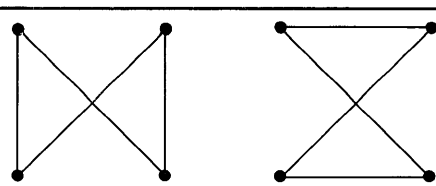


Try joining five equally spaced points.



Investigate different paths.

Investigate for other equally spaced points.



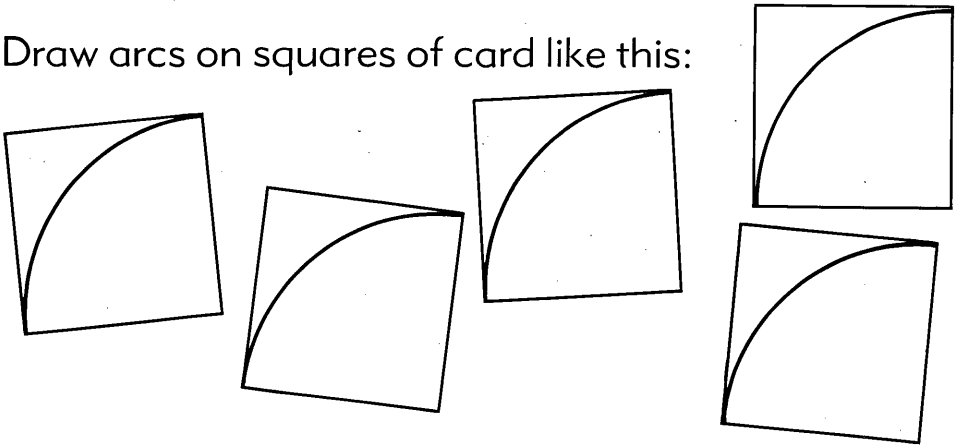
These are the same path.

ARC FORMS

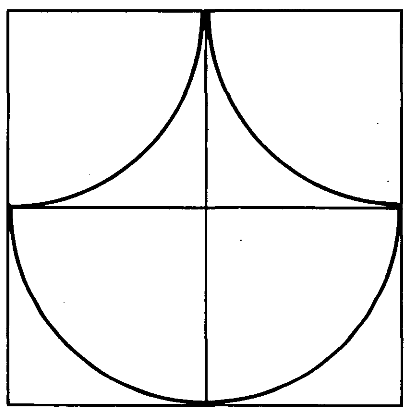
Card

Compasses

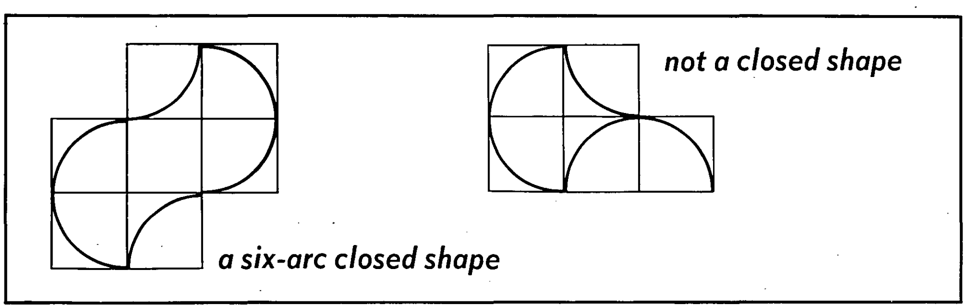
Draw arcs on squares of card like this:



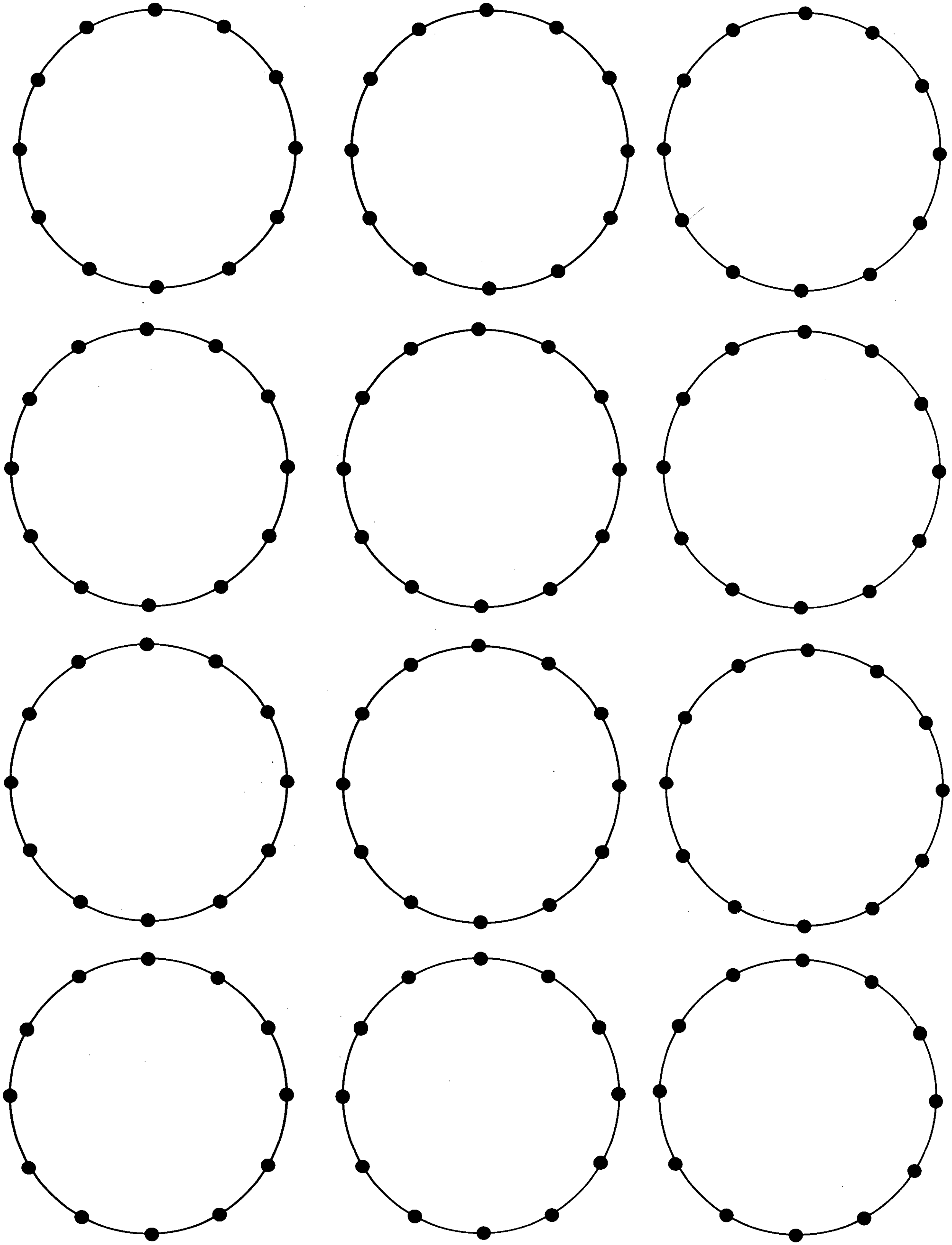
Each arc is a quarter circle.
Four arcs can be arranged to make a closed shape:



Investigate other four-arc closed shapes.
Investigate closed shapes with different numbers of arcs.



CIRCLES – 12 DOTS



DIGITS

ONE TO FOUR

(a) Use not more than one of each of the digits 1, 2, 3 and 4 in each expression;

(b) Use as many symbols as you like (e.g. $+$, $\times$, $-$) .... to make expressions for different numbers:

$\boxed{7} = 3 + 4$

$\boxed{17} = 21 - 4$

$\boxed{10} = 1 + 2 + 3 + 4$

$\boxed{73} = 32 + 41$

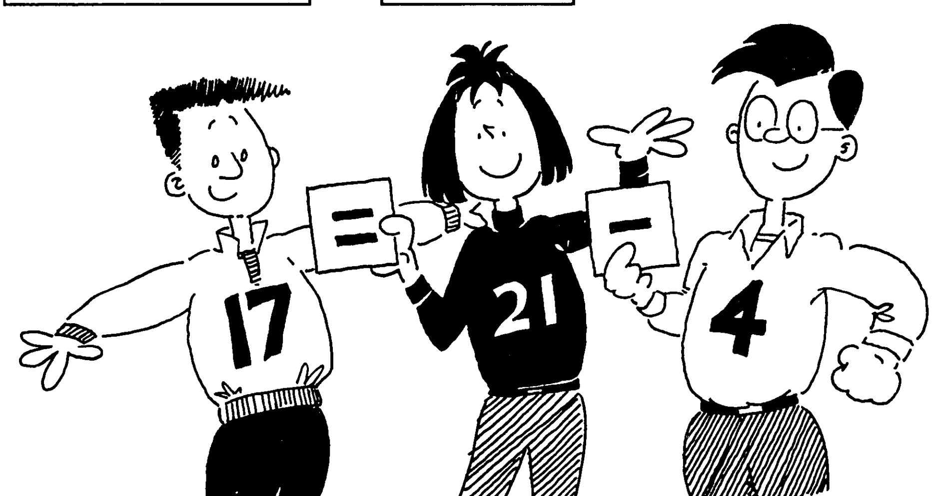
$\boxed{8} = 2 \times 4$

Can you find expressions for other numbers?

$1 = 4 - 3$
$2 = 3 - 1$
$3 = 1 + 2$
$4 = 1 + 3$
$5 = 4 + 2 - 1$
$6 =$

$\boxed{8} = 2^3$

$\boxed{2} = \sqrt{4}$



ROW SUMS

Number cards 0 – 9

Use four cards numbered 1, 2, 3 and 4.

1	2
3	4

Arrange them in a square grid.

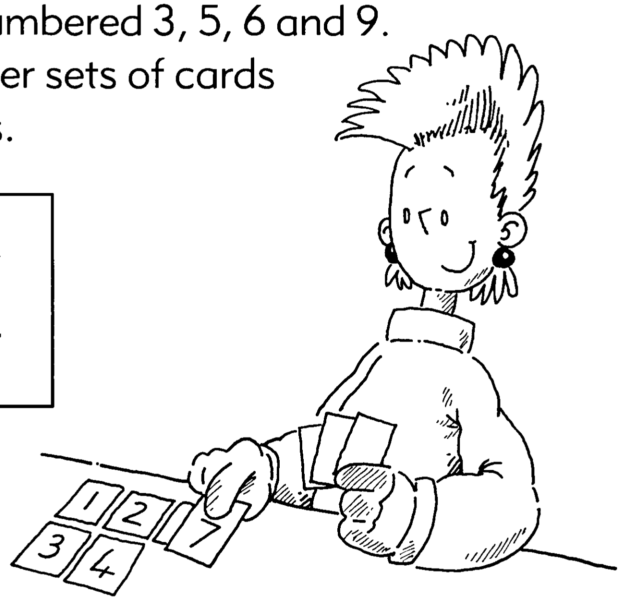
1	2	=	3
3	4	=	7

Find the row sums.

Try a different arrangement.
Investigate possible row sums.

Try using cards numbered 3, 5, 6 and 9.
Investigate for other sets of cards
and different grids.

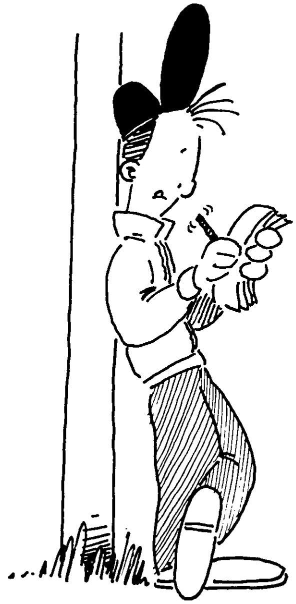
1	**3**	**5**	=	**9**
2	**4**	**6**	=	**12**



DIGITAL SUMS

The **digital sum** of 23 is 5.

$$23 \rightarrow 2 + 3 = 5$$



The **digital sum** of 84 is 3.

$$84 \rightarrow 8 + 4 = 12$$
$$12 \rightarrow 1 + 2 = 3$$

The digital sums of multiples of 3 are:

multiples of three	digital sums
3	3
6	6
9	9
12	3
15	6
18	9
21	3
24	6
27	9

What do you notice?
Investigate digital sums of other multiples.

DIGITS

60

SQUARE CARD SETS

Number cards 0 – 9

	square cards
[0] [1] [3] [4] [6] [7] [8] [9] The [2] and [5] can be combined to make the two-digit square number [2][5]. Place this to one side.	[2] [5]
[0] [1] [3] [6] [7] [8] [9] The single-digit square number can be produced from the card [4].	[4]
[0] [3] [7] [8] [9] From the remainder we can find [1][6]	[1] [6]
[0] [3] [7] [8] and then [9]	[9]

One **square card set** is 25, 4, 16, 9. Replace the ten cards and investigate other possible square card sets.

64, 9, ...

625

DIGITS

EQUATION SETS

Here are some **equation sets** using the digits:

1 to 4

$$2 + 3 = 4 + 1$$
$$4 - 3 = 2 - 1$$

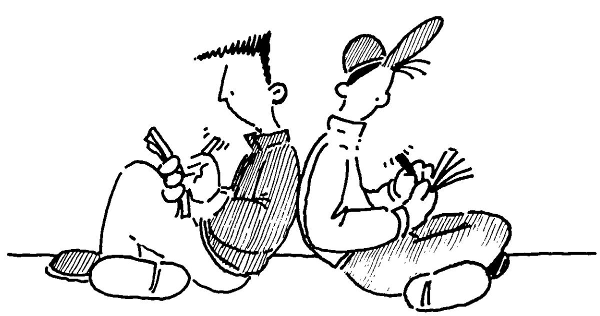
Each digit must be used once.
Can you find some more?

These equation sets use digits

1 to 6

$$6 \div 3 = 5 + 2 - 4 - 1$$
$$5 \times 4 \times 1 = (6 \times 3) + 2$$

Find some more.
Investigate some of your own equation sets.



ODDS AND EVENS

Number cards 1 – 9

Choose cards 2, 3, 4 and 7.
Arrange them on a 2 × 2 grid.
Find the **row sums**.

		row sums
2	3	5
4	7	11

Repeat using a different arrangement of 2, 3, 4 and 7.

Investigate arrangements which give:
(a) even row sums
(b) odd row sums
(c) an even and an odd row sum.

Find the column sums.

	2	3
	4	7
column sums	6	10

Investigate arrangements which give:
(a) even row and odd column sums
(b) odd row and odd column sums

Investigate for a different set of cards.

GRID SUMS

Squared paper

Copy this grid in which row sums and column sums are given.

		8
		6
10	4	

Write numbers on the grid which produce these row and column sums.

Here is a solution:

7	1	8
3	3	6
10	4	

How many other solutions can you find?

Investigate different solutions to these:

		9
		7
8	8	

		8
		6
9	5	

Invent some grids of your own and investigate possible solutions.

TARGET

Number cards 0 – 9

Operation cards +, −, ×, ÷

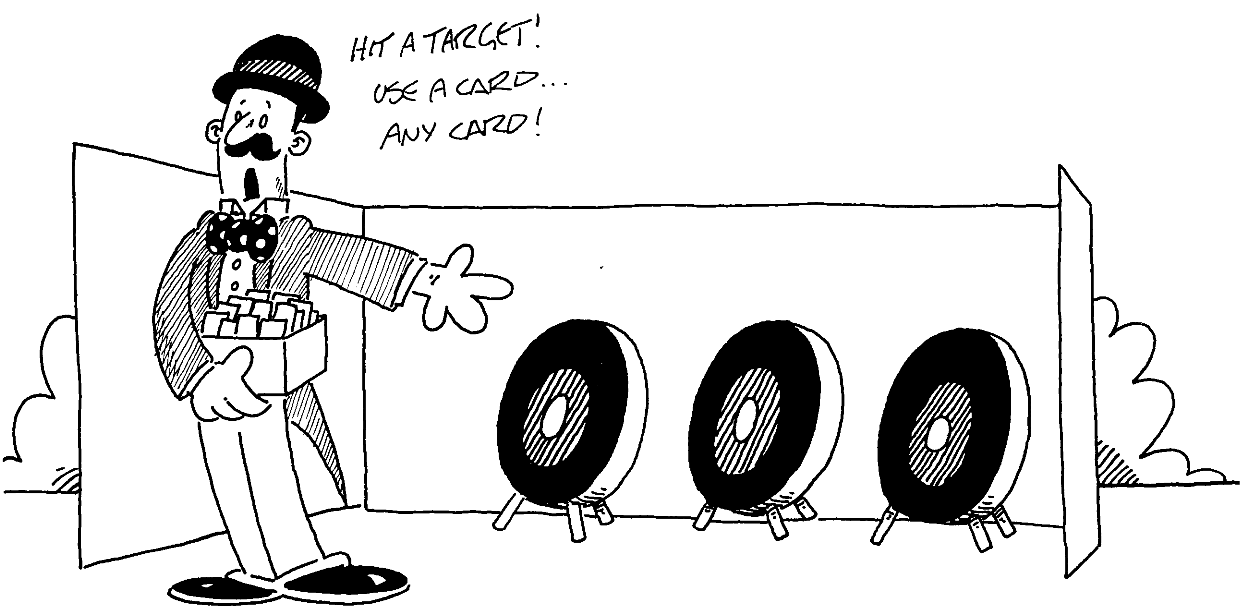
Choose a target. 7

Use the cards to hit the target.

9	−	2				uses three cards.
2	8	÷	4			uses four cards.
3	6	÷	4	−	2	uses six cards.

Find some other solutions.
Can you find a solution using more than six cards?

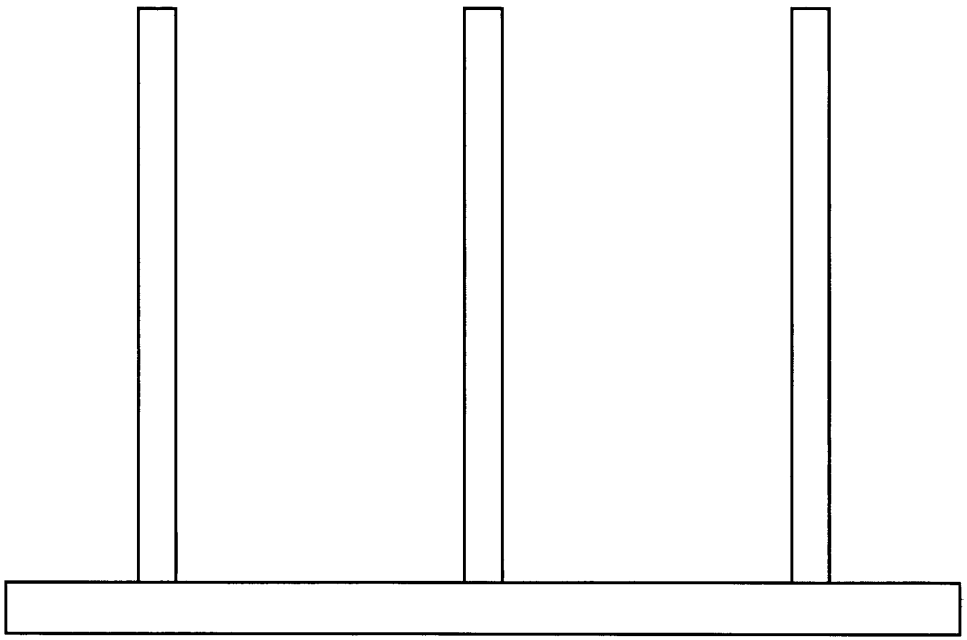
Investigate for other targets.



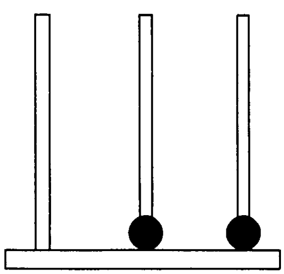
NUMBER II

SPIKE

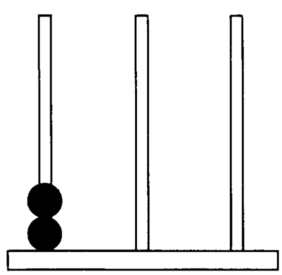
Counters or Spike abacus



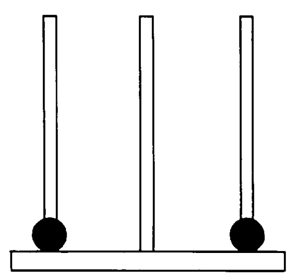
Here are three different numbers that can be made using two counters:



11



200



101

Can you find some more?

How many numbers can be made with three counters?

Investigate further.

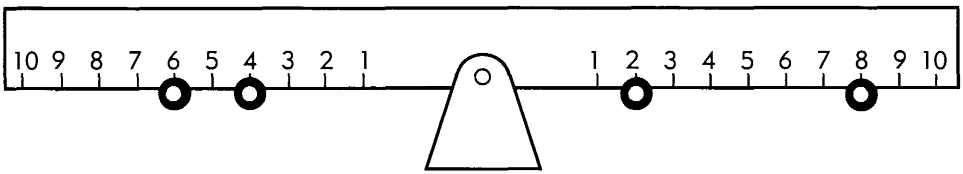
Find all the single-digit numbers
... then the two-digit numbers
... then the three-digit numbers.

BALANCE

Number balance

Washers

Washers have been placed on a number balance.

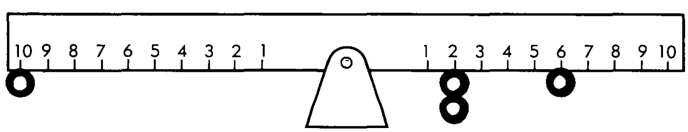


When each side has the same total you have **balanced numbers**.

$$6 + 4 = 2 + 8$$

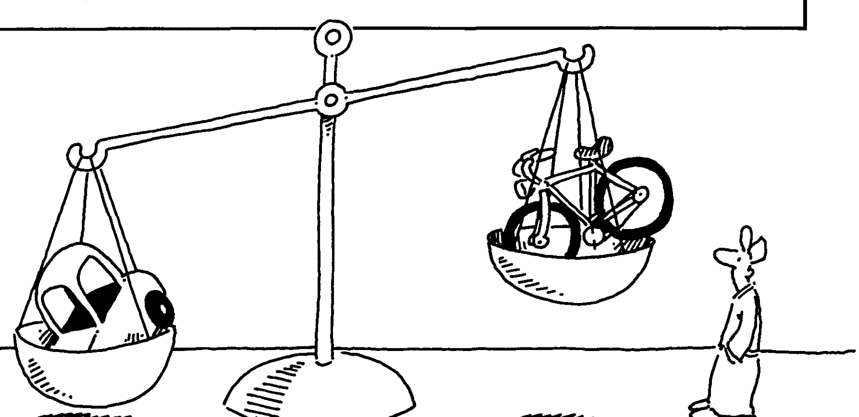
The washers can only be placed on **even** numbers.

Investigate different ways of balancing the washers.



$$\mathbf{10 = 2 + 2 + 6}$$

You do not have to place the same number of washers each side.
A number can have more than one washer.



THREE CARD TRICKS

Number cards 0-20

0 1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

Make a set of **three-card tricks** from the 21 cards.
The total for each trick must be 17.

Here is a set:

3 4 10   12 5 0   1 2 14

Investigate other sets for which each trick totals 17.

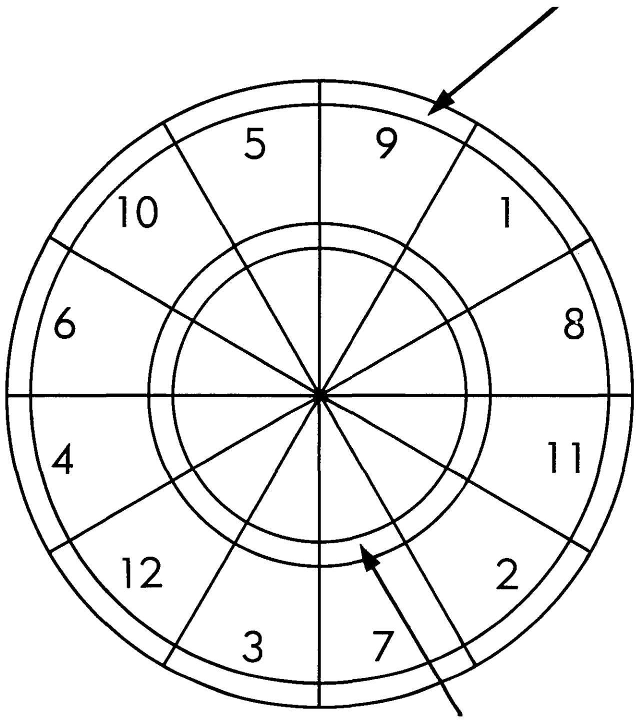
Investigate sets for other totals.

NUMBER II

DARTS

Double ring

a dart landing here scores **double** 9 = 18



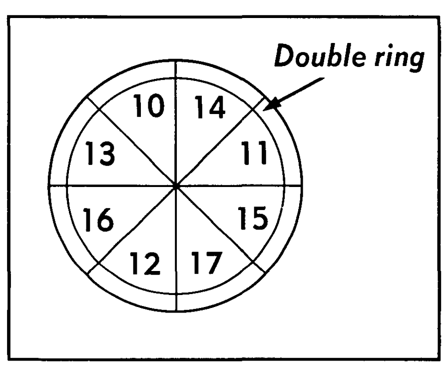
Treble ring

a dart landing here scores **treble** 11 = 33

What scores are possible with one dart?

Investigate possible scores with two or three darts.

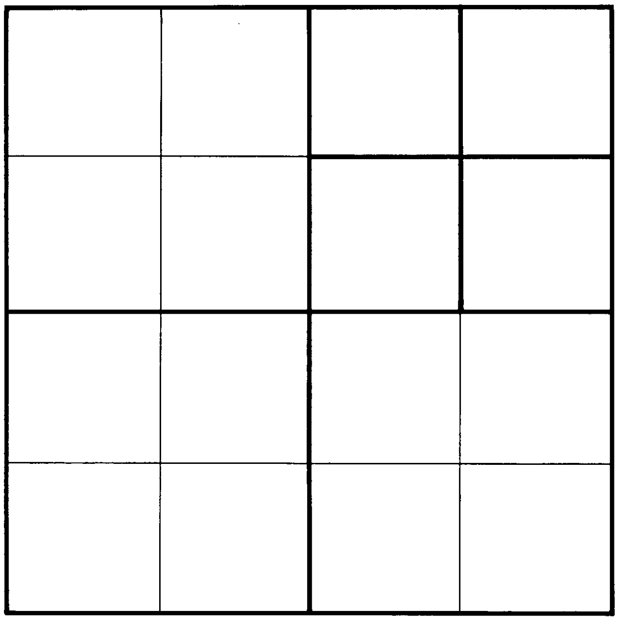
Invent different dartboards and investigate possible scores.



TIDY NUMBERS

Squared paper

Use a 4 × 4 grid.



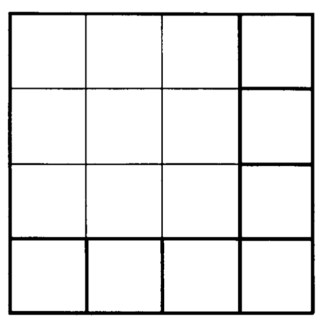
The grid can be divided into seven non-overlapping squares:

7 is a **tidy number**.

Find some more **tidy numbers** on the 4 × 4 grid.

Investigate tidy numbers on a 5 × 5 grid.

8 is a tidy number.



NEIGHBOURS

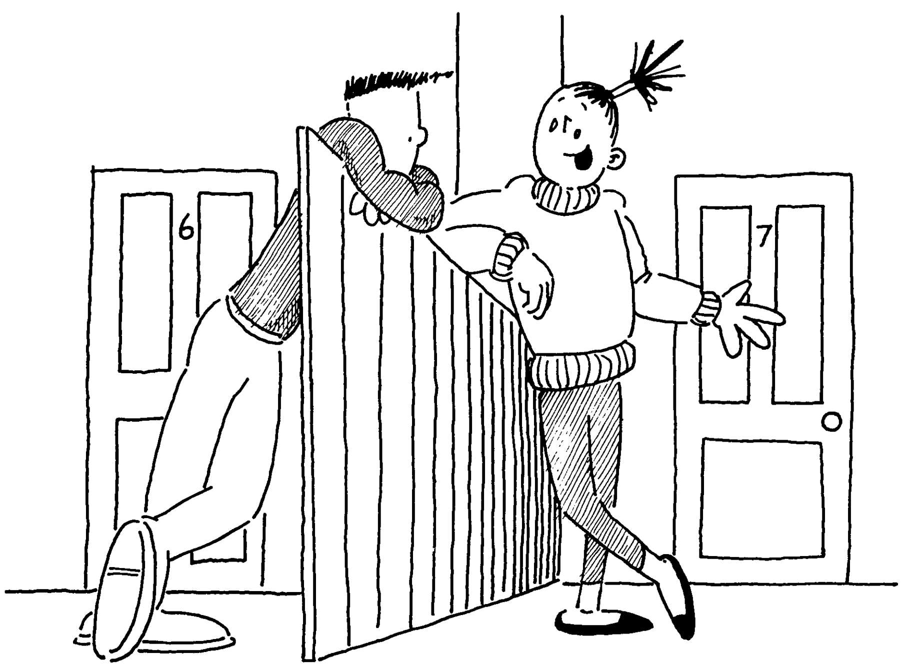
$$7 = 3 + 4$$
$$6 = 1 + 2 + 3$$
$$18 = 3 + 4 + 5 + 6$$

6, 7 and 18 can be expressed as the **sum of consecutive whole numbers**.

Investigate other numbers that can be expressed this way.

$$18 = 3 + 4 + 5 + 6 = 5 + 6 + 7$$

$$15 = 7 + 8 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5$$



THREES AND FIVES

6, 8, 11 and 15 can be obtained by adding **threes** and **fives**:

$$6 = 3 + 3$$
$$8 = 3 + 5$$
$$11 = 3 + 3 + 5$$
$$15 = 5 + 5 + 5$$

Investigate other totals obtained by adding **threes** and **fives.**
Try adding twos and sevens .

Investigate for other pairs of numbers.

$$\begin{aligned} 15 &= 5 + 5 + 5 \\ &= 3 + 3 + 3 + 3 + 3 \end{aligned}$$

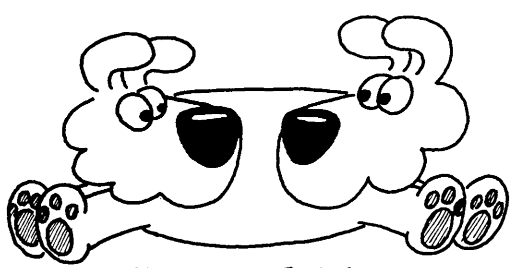
PALINDROMES

Choose a number	12	
Reverse the digits	21	
Add	33	**Palindromic**

Choose a number	38	
Reverse the digits	83	
Add	121	**Palindromic**

33 and 121 are **palindromic** – they read the same forwards and backwards.

Try another number	28	
Reverse the digits	82	
Add	110	
Reverse	011	
Add	121	**Palindromic**



12 and 38 required **one stage** to become palindromic.
28 required **two stages** to become palindromic.

Investigate other numbers.

1cm SQUARES

2 cm SQUARES

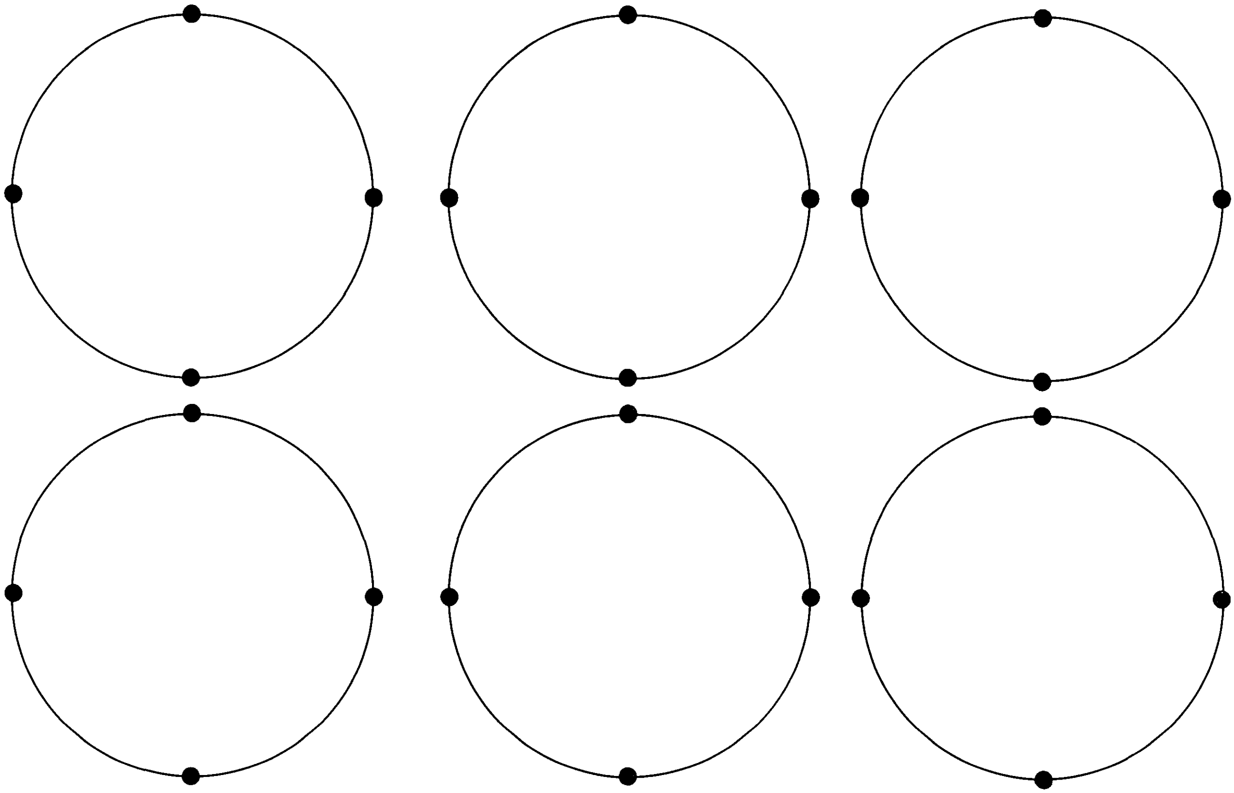
TABLE OF PRIMES

0–100	100–200	200–300
2	101	211
3	103	223
5	107	227
7	109	229
11	113	233
13	127	239
17	131	241
19	137	251
23	139	257
29	149	263
31	151	269
37	157	271
41	163	277
43	167	281
47	173	283
53	179	293
59	181	
61	191	
67	193	
71	197	
73	199	
79		
83		
89		
97		

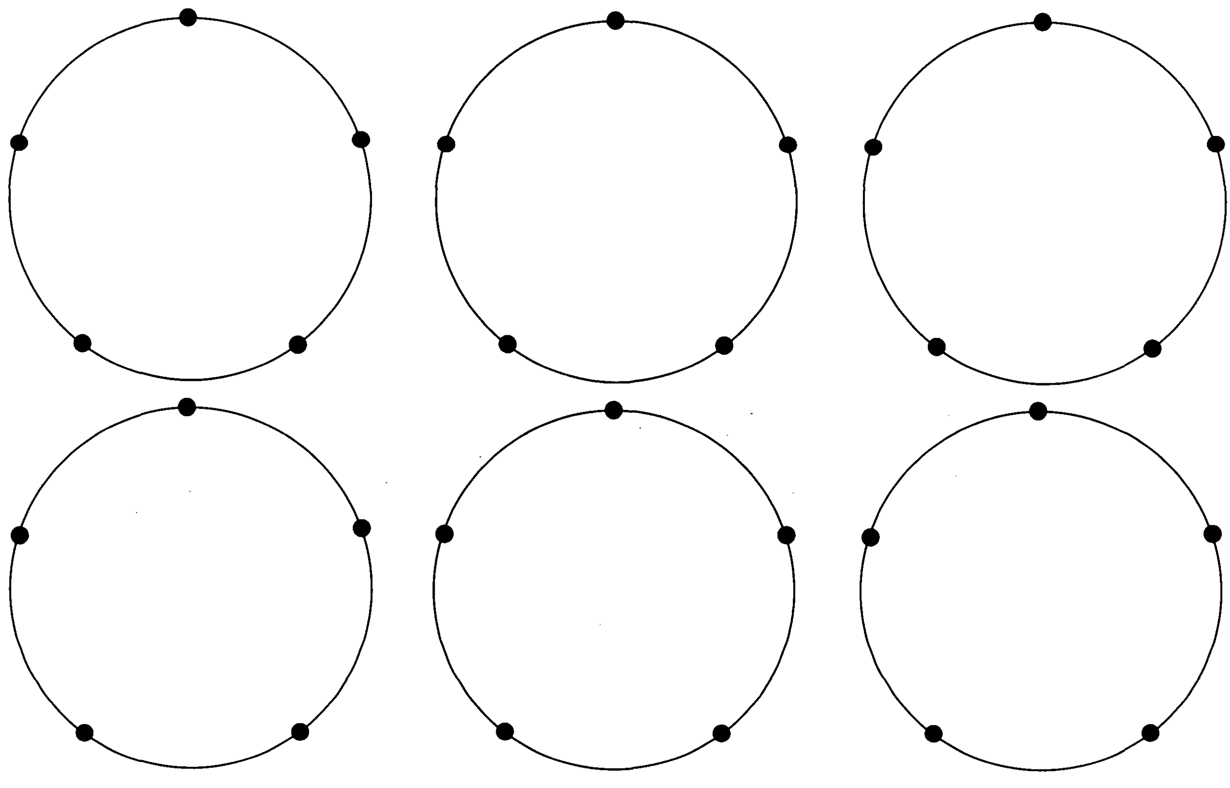
1cm SQUARE DOT

HEXAGON DOT

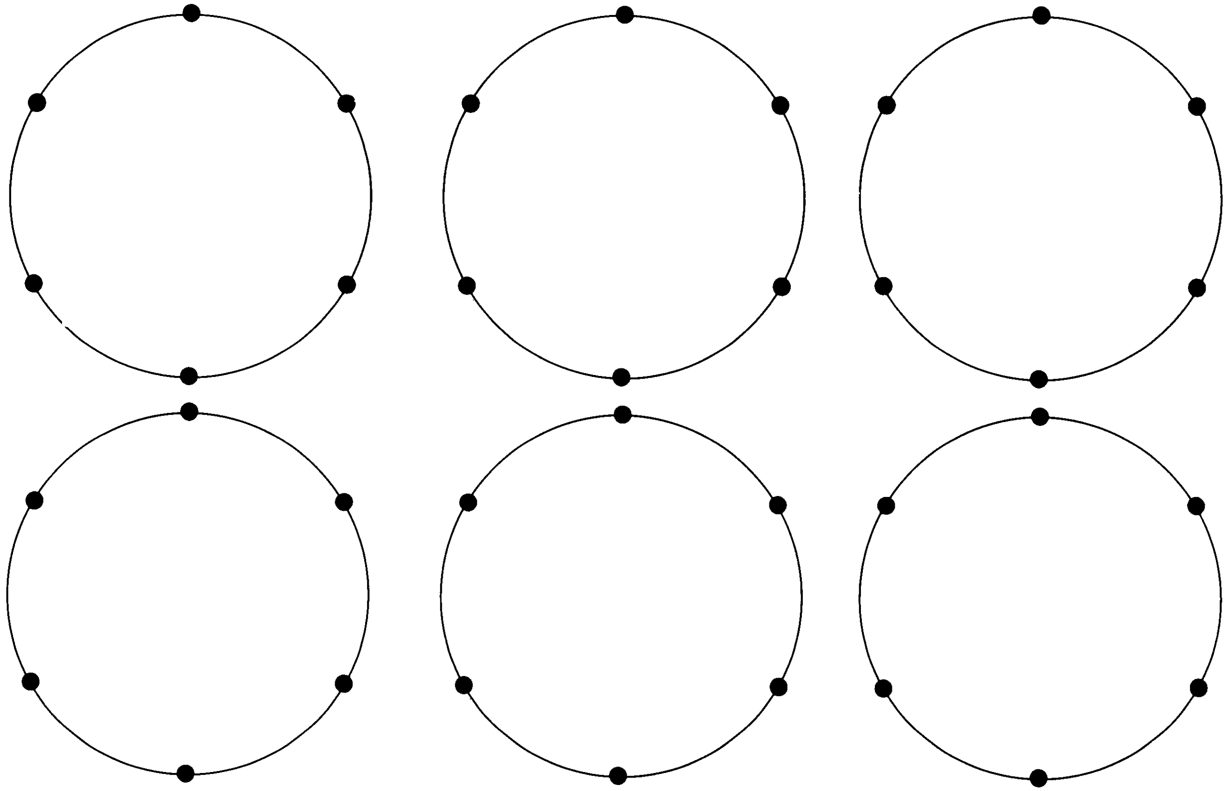
CIRCLES – 4 DOTS



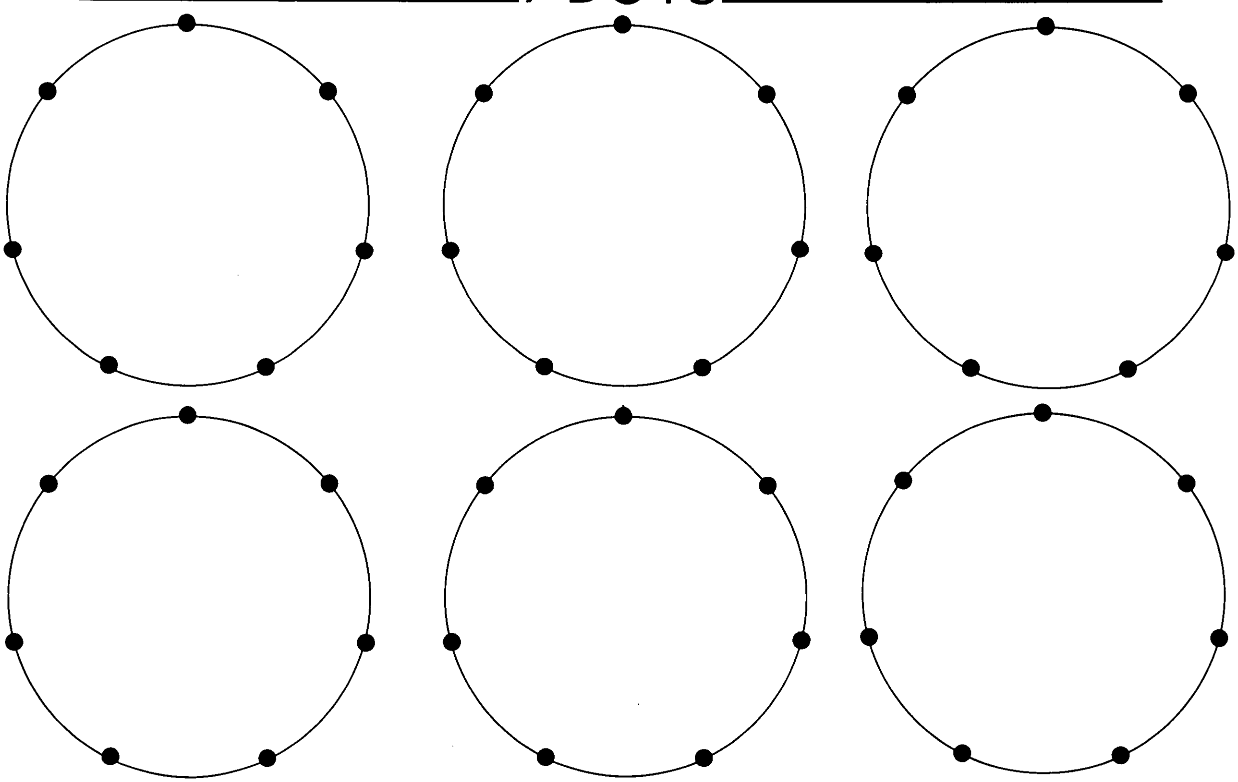
– 5 DOTS



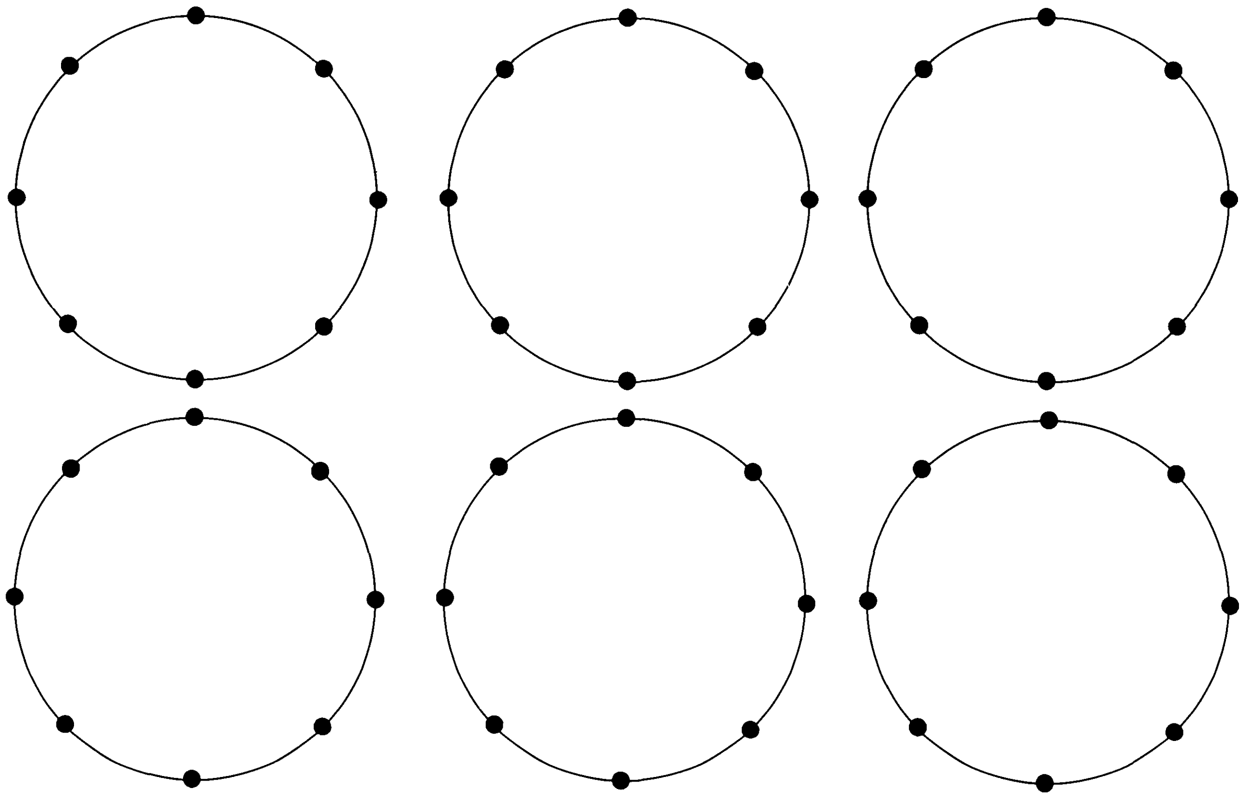
CIRCLES – 6 DOTS



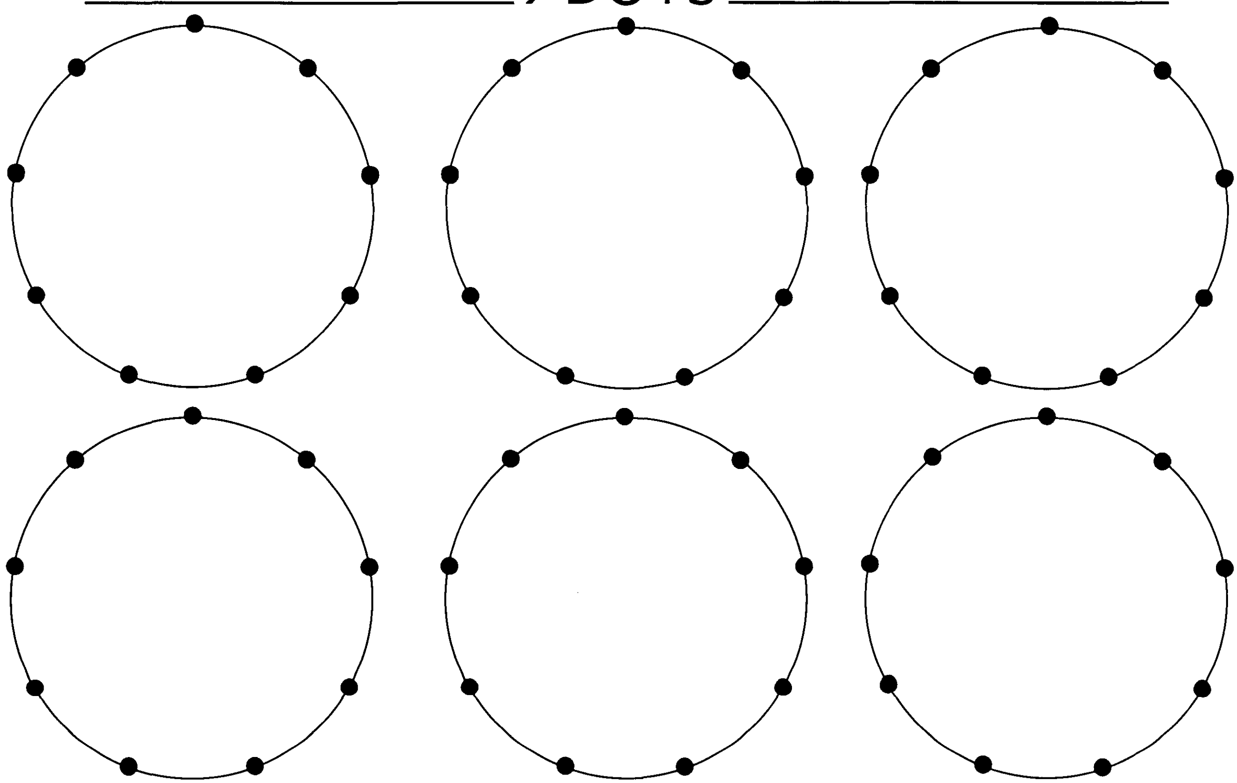
– 7 DOTS



CIRCLES – 8 DOTS



– 9 DOTS



2cm ISOMETRIC

1cm ISOMETRIC

M·A·T·H·S
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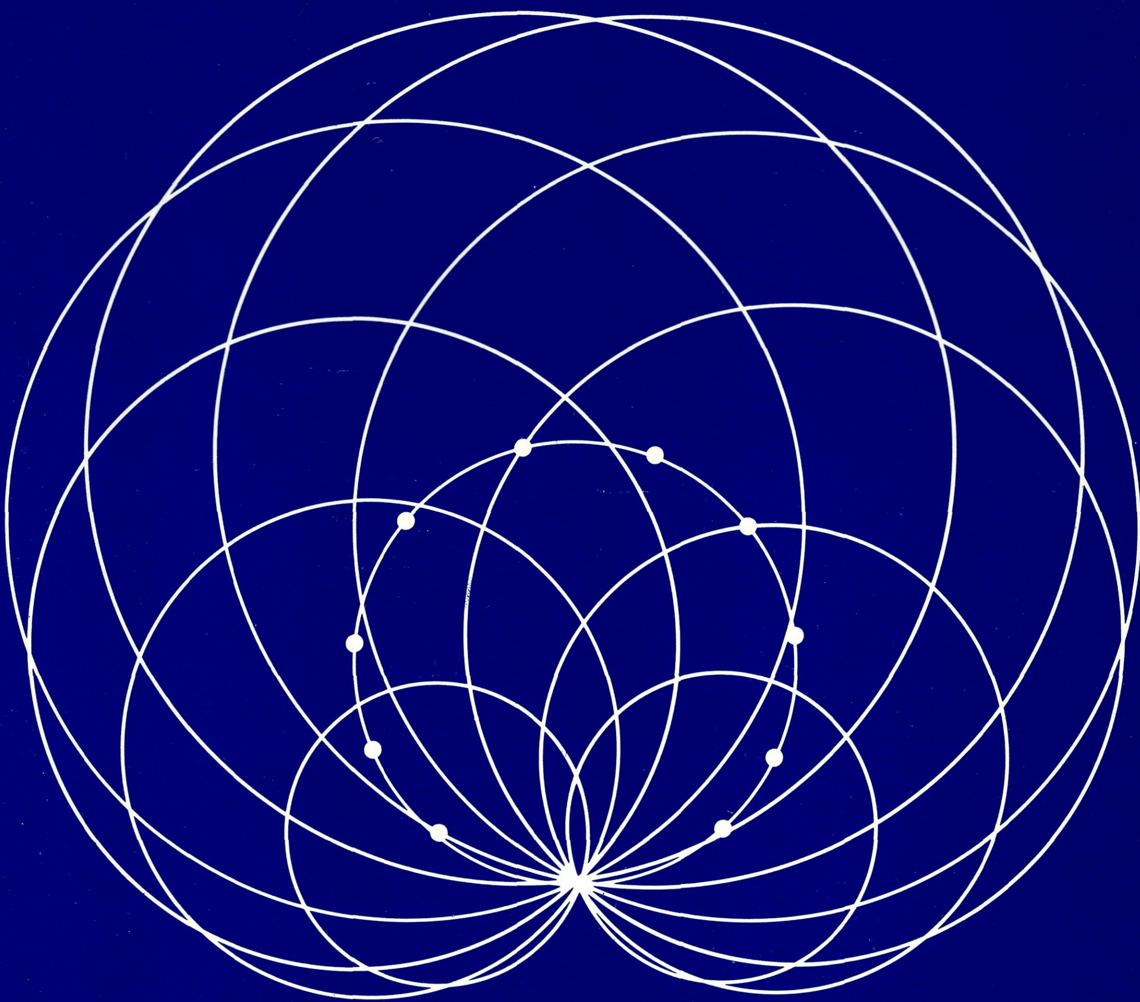
TEACHERS' GUIDE



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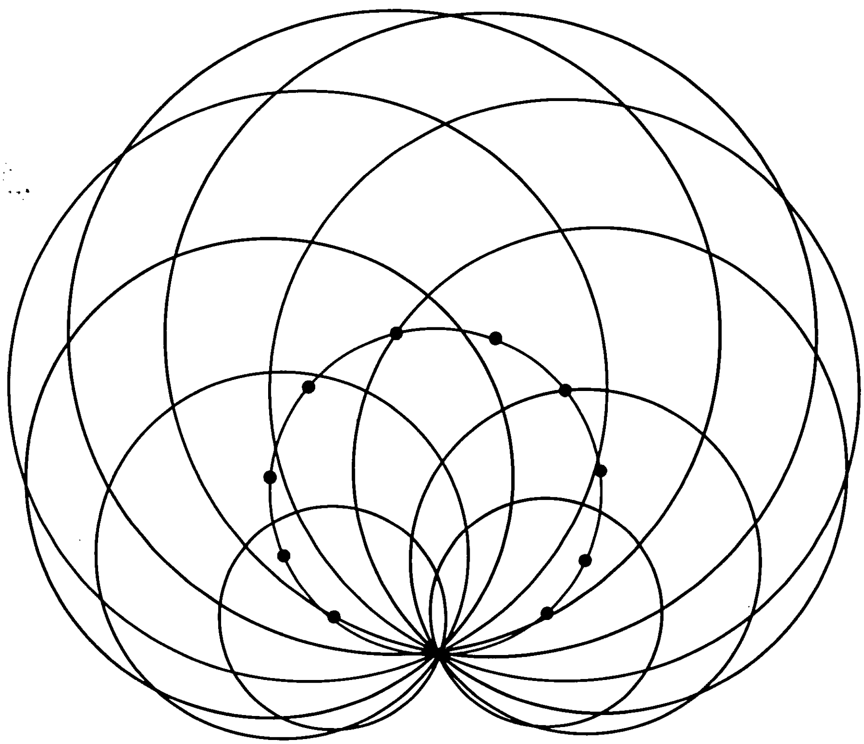

DAVID KIRKBY and PETER PATILLA

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